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SENSITIVITY ANALYSIS OF A RATE SENSOR WITH OBSERVER.(U)

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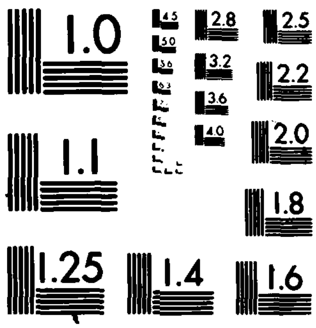

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Report EW-1-79

SENSITIVITY ANALYSIS OF A RATE SENSOR WITH OBSERVER

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January 1979

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A sensitivity study on a system composed of a rate sensor and its state estimating observer.

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BASIC NOMENCLATURE

A,B,...	Capital letters denote matrices.
a,b,c,... δ_i, ω..	Lower case letters denote scalers and constants.
$\underline{v}$, $\underline{x}$...	Underlining denotes vectors.
K_i	Denotes i^{th} element.
$E_{n,m}^{i,j}$	Denotes a nxm matrix of all zeros with a 1 in location i, j.
Δ_A	Characteristic equation of the sensor.
Δ_{A1}	Low frequency 2nd order term in Δ_A.
Δ_{A2}	High frequency 2nd order term in Δ_A.
Δ_F	Characteristic equation of the observer.

ABSTRACT

The automatic controls division of the Naval Air Development Center, (NADC) of Warminster, Pennsylvania, is currently investigating the use of the Model 8160 rate sensor offered by the inertial division of Systron Donner, Concord, California. This rate sensor appears to have the potential for reduced maintenance cost over current models, however, its output response has zero transmissibility at zero frequency and NADC wanted to determine if this could be altered through use of a filter.

An observer was designed to function as a state estimator for the rate sensor. Once the system states were available, the output of the observer was modeled after a desirable response. The resulting observer output performance exceeded expectations.

This report is a sensitivity study of the effect of all the rate sensor and observer parameters on the generated observer output.

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1.0 INTRODUCTION

1.1 Background

The automatic controls division of the Naval Air Development Center, (NADC) of Warminster, Pennsylvania, is currently investigating the use of the Model 8160 rate sensor offered by the inertial division of Systron Donner, Concord, California. Appendix A is the data sheet on the rate sensor. This rate sensor appears to have the potential for reduced maintenance cost over current models, however, its output response does not meet the requirements of NADC.

In a previous study, [1] an observer was designed to function as a state estimator and then the states were recombined so as to achieve a desirable output.

This report is a continuation of the previous one. The objective is to investigate the sensitivity of the generated output as a function of the rate sensor parameters and of the observer parameters.

2.0 SYSTEM EQUATIONS

2.1 Rate Sensor Equations

The rate sensor transfer function from Systron Donner, Appendix A, is

$$\frac{e_o}{\dot{\theta}} = 0.075\left(\frac{s^2}{s^2+.303s+.0656}\right)\left(\frac{15600}{s^2+175s+15600}\right) \qquad (2.1.1)$$

or

$$\frac{e_o}{\dot{\theta}} = \frac{1170s^2}{s^4+175.3s^3+15653s^2+4738.4s+1023.4} \qquad (2.1.2)$$

Referring to the frequency response curve on the data sheet in Appendix A, the low frequency washout is due to the loop Systron Donner added to reduce drift, and is probably adjustable. The high frequency corner is due to the sensor itself.

In order to deal with the parameters, let ω_1 and δ_1 represent the low frequency parameters, ω_2 and δ_2 represent the high frequency parameters, and a be the forward path gain as:

$$\frac{e_o}{\dot{\theta}} = \frac{as^2}{(s^2+2\delta_1\omega_1 s+\omega_1^2)(s^2+2\delta_2\omega_2 s+\omega_2^2)} \qquad (2.1.3)$$

or in more general terms:

$$\frac{e_o}{\dot{\theta}} = \frac{as^2}{\Delta_A} = \frac{as^2}{\Delta_{A1}\Delta_{A2}}$$

Now multiply the terms to get the general form:

$$\frac{e_o}{\dot{\theta}} = \frac{as^2}{s^4+es^3+ds^2+cs+b} \tag{2.1.4}$$

Clearly the coefficients b, c, d and e are functions of δ_1, ω_1, δ_2 and ω_2.

The transfer function of equation (2.1.4) may be written in phase variable canonical form [2] as:

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -b & -c & -d & -e \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ a \end{bmatrix} \dot{\theta} \tag{2.1.5}$$

$$e_o = [0 \quad 0 \quad 1 \quad 0] \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

or

$$\dot{\underline{q}} = A\underline{q} + B\theta$$
$$e_o = C_3\underline{q}$$

The $\underline{q}$ states are phase variable states and are defined as $\dot{q}_1 = q_2$, $\dot{q}_2 = q_3$, etc., the rate sensor output, e_o is a scaler signal.

The signal $\dot{\theta}$ is not available as an input, but θ is, and so equation 2.1.4 may be written in terms of θ as

$$\frac{e_o}{\theta} = \frac{as^3}{s^4+es^3+ds^2+cs+b} \tag{2.1.6}$$

This transfer function has a set of states different from the q states. Let these be the x states so that the phase variable canonical state equations become:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -b & -c & -d & -e \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \theta \tag{2.1.7}$$

$$e_o = [0 \quad 0 \quad 0 \quad 1] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

or

$$\underline{\dot{x}} = A\underline{x} + B\theta$$

$$e_o = C_4\underline{x}$$

Note that the A & B matrix are the same but in one case e_o is equal to the q_3 state and in the other e_o is equal to the x_4 state. This is a consequence of the phase variable definition of states. It is seen that the relation between the states is $\underline{\dot{x}} = \underline{q}$.

2.2 Observer Equations

The signal $\dot{\theta}$ is not available and so the observer must use θ as its input reference, hence the observer is defined [3,4,5] as

$$\dot{\underline{z}} = F\underline{z} + B\theta + \underline{K}e_o \qquad (2.2.1)$$

where F is unknown, B is the same as in 2.1.7, e_o is the rate sensor output, and $\underline{K}$ is the vector of observer gains

$$\underline{K} = \begin{bmatrix} K_1 \\ K_2 \\ K_3 \\ K_4 \end{bmatrix}$$

If the rate sensor is defined by equation (2.1.5), the error between the sensor states and observer states is $\underline{\varepsilon} = \underline{q}-\underline{z}$, or

$$\dot{\underline{\varepsilon}} = \dot{\underline{q}} - \dot{\underline{z}} = A\underline{q} + B\dot{\theta} - F\underline{z} - B\theta - \underline{K}e_o \qquad (2.2.2)$$

A problem arises in that θ and $\dot{\theta}$ can not be compared; hence it is necessary to use equation (2.1.17) to define the rate sensor [1]. Using (2.1.7), $\underline{\varepsilon} = \underline{x} - \underline{z}$ and so

$$\dot{\underline{\varepsilon}} = \dot{\underline{x}} - \dot{\underline{z}} = A\underline{x} + B\theta - F\underline{z} - B\theta - \underline{K}e_o \qquad (2.2.3)$$

Using e_o from (2.1.7) gives

$$\dot{\underline{\varepsilon}} = \dot{\underline{x}} - \dot{\underline{z}} = A\underline{x} - \underline{K}C_4\underline{x} - F\underline{z} \qquad (2.2.4)$$

If $F = A - \underline{K}C_4$, then

$$\underline{\varepsilon} = (A - \underline{K}C_4)\underline{x} - F\underline{z}$$

or

$$\dot{\underline{\varepsilon}} = F\underline{\varepsilon}$$

which converges if the eigenvalues of F are all negative, and the $\underline{K}$ vector is a set of arbitrary gains used to ensure this.

Hence, the observer must be determined in terms of the $\underline{x}$ states; however, once the observer dynamics are defined it makes no difference which states are used from thereon.

The auxiliary output, or generated output [1] is

$$\hat{e} = Mz = [0 \quad m_2 \quad m_3 \quad m_4] \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} \qquad (2.2.5)$$

$$= [0 \quad .87 \quad 4.04 \quad 13.33]\underline{z}$$

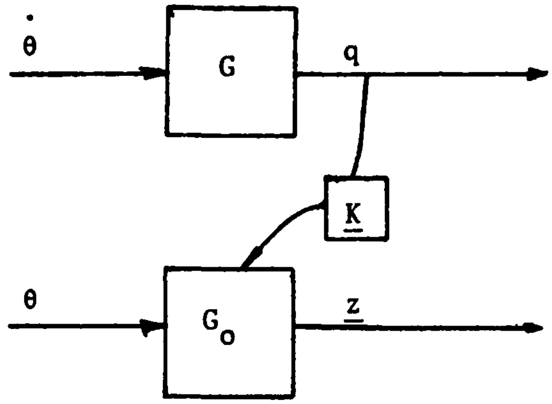
Therefore, the observer matrix equations are:

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & -K_1 \\ 0 & 0 & 1 & -K_2 \\ 0 & 0 & 0 & 1-K_3 \\ -b & -c & -d & -(e+K_4) \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ a \end{bmatrix} \theta + \begin{bmatrix} K_1 \\ K_2 \\ K_3 \\ K_4 \end{bmatrix} x_4 \qquad (2.2.6)$$

$$\hat{e} = [0 \quad m_2 \quad m_3 \quad m_4]\underline{z}$$

Figure 2.2.1 illustrates the above relations. As shown, under ideal conditions, the signal $\dot{\theta}$ and the signal θ are related as $\theta = \int\dot{\theta}dt$. When this is true, then the analysis using the model is generally simpler. If the mathematical relation between $\dot{\theta}$ and θ fails, then the relation in the top of Figure 2.2.1 must be used.

(a) System

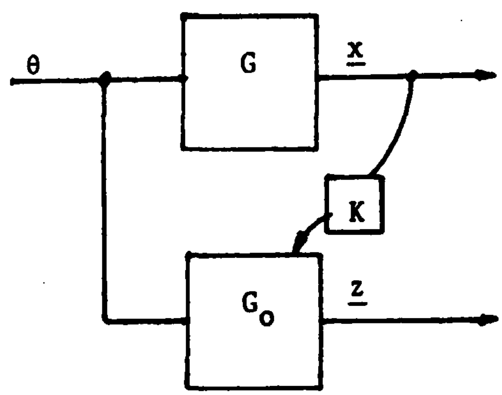


$$e_o = C_3(sI-A)^{-1}B\dot{\theta}$$

$$\dot{\underline{z}} = F\underline{z} + B\theta + \underline{K}e_o$$

$$\hat{e} = M(sI-F)^{-1}[B\theta + \underline{K}e_o]$$

(b) Model



$$e_o = C_4(sI-A)^{-1}B\theta$$

$$\underline{z} = F\underline{z} + B\theta + \underline{K}e_o$$

$$\hat{e} = M(sI-F)^{-1}[B\theta + \underline{K}e_o]$$

Figure 2.2.1 System Representation

2.3 System Equations

If the dynamics of the observer are augmented to those of the rate sensor, then a set of composite states may be defined as:

$$\dot{\underline{v}} = H\underline{v} + L\theta \qquad (2.3.1)$$

$$\hat{e} = D\underline{v}$$

where

$$\underline{v} = \begin{bmatrix} \underline{x} \\ \underline{z} \end{bmatrix}$$

so that

$$\dot{\underline{v}} = \begin{bmatrix} \underline{x} \\ \underline{z} \end{bmatrix} = \begin{bmatrix} A & 0 \\ KC_4 & F \end{bmatrix} \begin{bmatrix} \underline{x} \\ \underline{z} \end{bmatrix} + \begin{bmatrix} B \\ B \end{bmatrix} \theta \qquad (2.3.2)$$

$$\hat{e} = [0 \quad M] \begin{bmatrix} \underline{x} \\ \underline{z} \end{bmatrix}$$

or

$$\dot{\underline{v}} = \left[\begin{array}{cccc|cccc} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -b & -c & -d & -e & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & K_1 & 0 & 1 & 0 & -K_1 \\ 0 & 0 & 0 & K_2 & 0 & 0 & 1 & -K_2 \\ 0 & 0 & 0 & K_3 & 0 & 0 & 0 & 1-K_3 \\ 0 & 0 & 0 & K_4 & -b & -c & -d & -(e+K_4) \end{array}\right] v + \begin{bmatrix} 0 \\ 0 \\ 0 \\ a \\ 0 \\ 0 \\ 0 \\ a \end{bmatrix} \theta$$

$$\hat{e} = [\,0 \quad 0 \quad 0 \quad 0 \;\vdots\; 0 \quad m_2 \quad m_3 \quad m_4\,]\,\underline{v}$$

Equations (2.3.2) are the full set of state equations that describe the dynamics of the rate sensor and observer.

The solution to equation (2.3.1) is

$$\underline{v}(s) = (sI-H)^{-1}L\theta(s)$$

and

$$\hat{e}(s) = D(sI-H)^{-1}L\theta(s) \qquad (2.3.3)$$

In terms of equation (2.3.2), the $(sI-H)^{-1}$ matrix is

$$\left\{ sI - \begin{bmatrix} A & 0 \\ \underline{K}C_4 & F \end{bmatrix} \right\}^{-1} = \begin{bmatrix} sI-A & 0 \\ -\underline{K}C_4 & sI-F \end{bmatrix}^{-1} \qquad (2.3.4)$$

The inverse of (2.3.4) is

$$(sI-H)^{-1} = \begin{bmatrix} (sI-A)^{-1} & 0 \\ (sI-F)^{-1}\underline{K}C_4(sI-A)^{-1} & (sI-F)^{-1} \end{bmatrix} \qquad (2.3.5)$$

The A matrix was defined in equation (2.1.6) as

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -b & -c & -d & -e \end{bmatrix}$$

and so

$$(sI-A)^{-1} = \frac{1}{\Delta_A}\begin{bmatrix} s^3+es^2+ds+c & s^2+es+d & s+e & 1 \\ -b & s^3+es^2+ds & s^2+es & s \\ -bs & -cs-b & s^3+es^2 & s^2 \\ -bs^2 & -cs^2-bs & -ds^2-cs-b & s^3 \end{bmatrix} \qquad (2.3.6)$$

where Δ_A is the characteristic equation of the $(sI-A)^{-1}$ matrix and is

$$\Delta_A = s^4+es^3+ds^2+cs+b$$

$$= \Delta_{A1}\Delta_{A2} = (s^2+\delta_1\omega_1 s+\omega_1^2)\ (s^2+2\delta_2\omega_2 s+\omega_2^2)$$

Next, given the F matrix in (2.2.3) as

$$F = \begin{bmatrix} 0 & 1 & 0 & -K_1 \\ 0 & 0 & 1 & -K_2 \\ 0 & 0 & 0 & 1-K_3 \\ -b & -c & -d & -(e+K_4) \end{bmatrix} \tag{2.3.7}$$

then equation (2.3.8) on the next page may be found.

Knowing the relation (2.3.5), the only terms missing in $(sI-H)^{-1}$ are the $(sI-F)^{-1}\underline{K}C_4(sI-A)^{-1}$ terms. This is a 4x4 matrix and in general the terms are quite complex. Because it is encountered quite often, it is defined as

$$(sI-F)^{-1}\underline{K}C_4(sI-A)^{-1} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} \\ C_{21} & C_{22} & C_{23} & C_{24} \\ C_{31} & C_{32} & C_{33} & C_{34} \\ C_{41} & C_{42} & C_{43} & C_{44} \end{bmatrix} \tag{2.3.9}$$

It is found that the C_{24}, C_{34} and C_{44} terms are used quite often and so are written out for future reference:

$$\begin{aligned} C_{24} &= K_2s^6 + (K_3+K_2e)s^5+(K_4+K_3e+K_2d)s^4-K_1bs^3 \\ C_{34} &= K_3s^6+(K_3e+K_4)s^5+(-K_1b-K_2C)s^4-K_2bs^3 \\ C_{44} &= K_4s^6-(K_1b+K_2C+K_3d)s^5-(K_2b+K_3C)s^4-K_3bs^3 \end{aligned} \tag{2.3.10}$$

Additional terms will be found as needed.

Equation 2.3.8

let $h = e+K_4$, $J = K_3-1$

$$(sI-F)^{-1} = \frac{1}{\Delta_F}\begin{bmatrix} s^3+hs^2-(Jd+cK_2)s-Jc & s^2+(cK_1+h)s-Jd & s(k_1d+1)+h+K_2d+K_1c & -(s^2K_1+sK_2+J) \\ sK_2b+Jb & s^3+s^2h-(Jd+bK_1)s & s^2+s(h+K_2d)-K_1b & -(s^2K_2+sJ) \\ sJb & sJc+Jb & s^3+hs^2+(bK_1-cK_2)s-bK_2 & -(s^2J) \\ -s^2b & -(s^2c+sb) & -s^2d+sc+b) & s^3 \end{bmatrix}$$

$$\Delta_F = s^4+(e+K_4)s^3+[d(1-K_3)-cK_2-bK_1]s^2 + [c(1-K_3)-bK_2]s + b(1-K_3) = 0$$

Referring back to equations (2.3.3) and using (2.3.2) and (2.3.5), the solution of the composite system may be written as

$$\hat{e}(s) = [0 \quad M] \begin{bmatrix} (sI-A)^{-1} & 0 \\ & \\ (sI-F)^{-1}\underline{K}C_4(sI-A)^{-1} & (sI-F)^{-1} \end{bmatrix} \begin{bmatrix} B \\ \\ B \end{bmatrix} \qquad (2.3.11)$$

$\hat{e}(s)$ and $\theta(s)$ are scalers and so the remainder of the terms are the transfer function of the observer output with θ input. Expanding (2.3.11) gives

$$\hat{e}(s) = M\{(sI-F)^{-1}\underline{K}C_4(sI-A)^{-1} + (sI-F)^{-1}\}B\theta(s) \qquad (2.3.12)$$

The expression between the brackets is rather messy, however:

<u>Theorem</u>

$$(sI-A)^{-1} = (sI-F)^{-1}\underline{K}C_4(sI-A)^{-1} + (sI-F)^{-1} \qquad (2.3.13)$$

<u>Proof</u>

Post multiply both sides of (2.3.13) by (sI-A) to get

$$I = (sI-F)^{-1}\underline{K}C_4 + (sI-F)^{-1}(sI-A)$$

then premultiply both sides by (sI-F) to get

$$(sI-F) = \underline{K}C_4 + (sI-A)$$

But by definition, equation (2.2.4), $F - A-\underline{K}C_4$ and so

$$sI-F = sI-A+\underline{K}C_4$$

$$= sI-(A-\underline{K}C_4)$$

Therefore, equation 2.3.12 simplifies to

$$\hat{e}(s) = M(sI-A)^{-1}B\theta(s) \tag{2.3.14}$$

Of course this is not valid at all times as explained below.

2.4 Assumptions

As with any sensitivity study, the object is to determine the effect of each parameter independently of all others. Hence, the assumption is made that we know exactly the rate sensor parameters and that we design the observer with exactly the values desired. Then the effect as each parameter varies from its ideal value is investigated.

Along with this is the assumption that $\theta = \int\dot{\theta}dt$, the signals θ and $\dot{\theta}$ are assumed to have the proper mathematical relation. As seen with the Theorem of last section, if the θ - $\dot{\theta}$ relation holds it simplifies the analysis at times. The effect of variations between θ and $\dot{\theta}$ is also investigated.

In some cases the effect on the output changes depending upon where the parameter changes. That is, if the forward path gain of the rate sensor varies 10% from the ideal, the effect is different than if the forward path gain of the observer is off 10%.

2.5 Effect of the Observer Root Locations

The characteristic equation of the observer is Δ_F and is defined in equation (2.3.8), where it is seen that the roots of Δ_F are functions of the observer gains, $\underline{K}$. The observer is appended to the rate sensor and the total system

has the characteristic equation $\Delta_A \Delta_F$, where Δ_A is the characteristic equation of the sensor. Therefore, the observer does not affect the roots of the sensor, but adds its own roots to the overall system. Since the roots of Δ_F depend upon the observer gains $\underline{K}$, then changing the gains changes the roots.

In this case the observer is functioning as a state estimator, that is, it is generating the rate sensor states. The observer gains are arbitrary to the extent that the roots of Δ_F must all have negative real parts. After this, the magnitude of the real parts of the roots of Δ_F determine how fast the observer states converge and track the sensor states. As found in [1], placing the negative real part of the observer roots between -2 and -10 appears to give reasonable results.

From the standpoint of the observer tracking the sensor states, the observer root locations do not matter once initial condition transients die out. That is, the observer root locations determine the rate of convergence to the plant states initially, but they all track the same from then on. Observer roots with a negative real part of -2 converge to the sensor states in approximately two seconds, and track from then on. Two seconds compared to the start up time of the airplane means nothing.

On the other hand, refer to Figure 2.2.1(a). Note that the connection between the system and the observer is through the gains $\underline{K}$. If the gains $\underline{K} \rightarrow o$, then the output $\hat{e}$ is totally dependent on the observer parameters, which reduces to the model equations of the sensor. On the other hand, as the gains $\underline{K}$ become large, dependence upon the model parameters decreases.

In general, as the $\underline{K}$ gains increase, the real part of the roots of the observer become more negative, but they must do this according to Δ_F of equation (2.3.8) in order to have a stable system. To insure this, the roots of Δ_F are generally first placed and the $\underline{K}$ gains then calculated.

Throughout the study, in most cases, the observer gains had effects on sensitivity. In all cases, it was assumed that all four roots of Δ_F were at -2, -5, or -10 on the real axis, and thus on all figures, the curves are identified with a 2, 5 or 10. The corresponding gains are [1]:

Observer Gains

Root Locations		K_1	K_2	K_3	K_4
	-2	.0253	.0411	.984	-167.3
	-5	-1.64	2.34	.39	-155.3
	-10	-42.5	41.3	-8.77	-135.3

3.0 SENSITIVITY EQUATIONS

3.1 Background

Sensitivity, as first defined by Bode and as defined in most basic automatic controls texts [6,7] is currently called the logrithmetic sensitivity ratio. According to this, the sensitivity of P(s) with respect to Q(s) is

$$S_Q^P = \frac{\partial P(s)/P(s)}{\partial Q(s)/Q(s)} = \frac{\partial \ell P(s)}{\partial \ell Q(s)}$$

or

$$S_Q^P = \frac{Q(s)}{P(s)} \cdot \frac{\partial P(s)}{\partial Q(s)}$$

Once S_Q^P has been found, in terms of a transfer function, then given a percentage change in Q, $\frac{\Delta Q}{Q} \simeq \frac{\partial Q}{Q}$ then the percent change in P is

$$\frac{\partial P}{P} \simeq \frac{\Delta P}{P} = \frac{\Delta Q}{Q} \cdot S_Q^P$$

This works well for transfer functions of relatively low order.

If S_Q^P is computed, only, as a time solution, note that the solution, P is in the denominator of S_Q^P . In addition, the above does not lend itself to matrix equations in the time domain.

3.2 Logarithmic Sensitivity Trajectory Function

Therefore, most current work in the area of parameter sensitivity studies employ the sensitivity trajectory function

$$q = \frac{\partial P}{\partial Q}$$

or the logarithmic sensitivity trajectory function [8-16]

$$\mu_Q^P = Q\frac{\partial P}{\partial Q} = \frac{\partial P}{\partial Q/Q} \qquad (3.2.1)$$

where Q is a parameter. (Actually, μ_Q^P is a sensitivity coefficient [8].)

The definition of sensitivity as given by equation (3.2.1) is the one used in this report.

In equation (3.2.1), since

$$\frac{\partial P}{\partial Q} \simeq \frac{\Delta P}{\Delta Q}$$

the deviation in P, ΔP due to a percent change in parameter Q, $\Delta Q/Q$ is

$$\mu_Q^P = Q\,\frac{\Delta P}{\Delta Q}$$

or

$$\Delta P = \frac{\Delta Q}{Q}\ \mu_Q^P \qquad (3.2.2)$$

Thus, given a percent deviation in Q, this value is multiplied times the value of μ_Q^P to find the deviation in P. Note that since Q is a parameter, then $\Delta Q/Q$ is in general just a number, 10%. Therefore, the curve μ_Q^P is actually the curve of deviation in P; only the magnitude needs to be altered. Also, note that equation (3.2.1) is easy to use with matrix equations and time solutions.

Equation (3.2.1) is related to S_Q^P from

$$S_Q^P = \frac{\Delta P/P}{\Delta Q/Q}$$

and

$$\Delta P = P\,\frac{\Delta Q}{Q}\ S_Q^P$$

so that

$$S_Q^P = \frac{1}{P}\,\mu_Q^P \qquad (3.2.3)$$

3.3 Sensitivity Equations

The rate sensor dynamics augmented with the observer dynamics were written as

$$\dot{\underline{v}} = H\underline{v} + L\theta \qquad (3.3.1)$$

$$\hat{e} = D\underline{v}$$

The solution of this was

$$\underline{v}(s) = (sI-H)^{-1}L\theta(s) \qquad (3.3.2)$$

and $$\hat{e}(s) = D(sI-H)^{-1}L\theta(s) \qquad (3.3.3)$$

Since $\hat{e}$ and θ are scalers, then (3.3.3) may be written as

$$\frac{\hat{e}(s)}{\theta(s)} = D(sI-H)^{-1}L \qquad (3.3.4)$$

and the right side of the equal sign is the transfer function of the total system from input θ to observer output, $\hat{e}$. Let the elements of the D matrix be denoted as d_{ij}, and h_{ij} and l_{ij} be elements of the H and L matrices respectively, that is

$$H = [h_{ij}]$$

$$D = [d_{ij}]$$

$$L = [l_{ij}]$$

Let $E_{n,n}^{i,j}$ be a matrix of order nxm where $E_{n,m}^{i,j}$ has zeros at every entry except the entry at row i, column j, which is unity. Thus,

$$E_{2,3}^{2,1} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

In the case that $E^i_{n,m}$ is a row matrix or a vector, it will have only one superscript.

To find the sensitivity of $\hat{e}$ with respect to an element of the output matrix, D, in equation (3.3.4), it is necessary to find

$$\mu^{\hat{e}}_{d_i} = \frac{\partial \hat{e}}{\partial d_i / d_i} = \frac{d_i \; \partial e}{\partial d_i} \tag{3.3.5}$$

From (3.3.4) it is seen that

$$\frac{\partial \hat{e}}{\partial d_i} = E^i_{1,n}(sI-H)^{-1}L\theta \tag{3.3.6}$$

and so

$$\mu^{e}_{d_i} = d_i \, E^i_{1,n} \; (sI-H)^{-1}L\theta \tag{3.3.7}$$

To find the sensitivity of $\hat{e}$ with respect to an element of the L matrix, first from 3.3.4

$$\frac{\partial \hat{e}}{\partial \ell_j} = D(sI-H)^{-1}E^j_{n,1}\theta \tag{3.3.8}$$

then

$$\mu^{\hat{e}}_{\ell_j} = \ell_j D(sI-H)^{-1}E^j_{n,1}\theta \tag{3.3.9}$$

In order to find the sensitivity of $\hat{e}$ with respect to an element of the H matrix, first from 3.3.4

$$\frac{\partial e}{\partial h_{ij}} = D(sI-H)^{-1} \frac{\partial H}{\partial h_{ij}}(sI-H)^{-1}L\theta \tag{3.3.10}$$

then

$$\mu^{\hat{e}}_{h_{ij}} = h_{ij}D(sI-H)^{-1} \frac{\partial H}{\partial h_{ij}} (sI-H)^{-1}L\theta \tag{3.3.11}$$

Equations (3.3.7), (3.3.9) and (3.3.11) are each expanded and treated independently in later sections.

4.0 TOTAL SYSTEM RESPONSE

4.1 Frequency and Time Responses

The combined sensor and observer state equations are

$$\underline{\dot{v}} = H\underline{v} + L\theta$$

$$\hat{e} = D\underline{v}$$

with the s-domain solution

$$\hat{e}(s) = D(sI-H)^{-1}L\theta(s) \tag{4.1.1}$$

Using equations (2.3.2) and (2.3.5) gives

$$\frac{\hat{e}(s)}{\theta(s)} = [0 \quad M] \begin{bmatrix} (sI-A)^{-1} & 0 \\ (sI-F)^{-1}\underline{K}C_4(sI-A)^{-1} & (sI-F)^{-1} \end{bmatrix} \begin{bmatrix} B \\ B \end{bmatrix} \tag{4.1.2}$$

or

$$\frac{\hat{e}(s)}{\theta(s)} = M\{ (sI-F)^{-1}\underline{K}C_4(sI-A)^{-1} + (sI-F)^{-1}\} B \tag{4.1.3}$$

Equation (4.1.3) may be reduced by use of the theorem, equation (2.3.13) to:

$$\hat{e} = M(sI-A)^{-1}B\theta \tag{4.1.4}$$

The coefficients of M were deived from the low frequency second order terms of equation (2.1.3) [1], and so equation (4.1.4) reduces to

$$\frac{\hat{e}(s)}{\theta(s)} = \frac{s\omega_2^{\,2}}{s^2+2\delta_2\omega_2 s+\omega_2^{\,2}} \tag{4.1.5}$$

This was the response designed for in [1] and is a simple second order response. Written in terms of $\dot{\theta}$, (4.1.5) becomes

$$\frac{\hat{e}(s)}{\dot{\theta}(s)} = \frac{\omega_2^{\,2}}{s^2+2\delta_2\omega_2 s+\omega_2^{\,2}} \tag{4.1.6}$$

Figure 4.1.1 illustrates the frequency domain solution of equation (4.1.1) with a sine wave input at $\dot{\theta}$. This is seen to be the same response as one would get from equation (4.1.6).

Figure 4.1.2 illustrates the time response of the output $\hat{e}$ from the solution of the state equations (2.3.1) with a unit step input at $\dot{\theta}$. Again, this is the response one would expect from equation (4.1.6).

If the initial values of the observer and the sensor are not the same, then there is a period of initial condition transients that must decay. Figure 4.1.3 illustrates the response of $\hat{e}$ due to different initial states of the sensor and observer. The input $\dot{\theta}$ was a square wave of period 1 second and unity magnitude. Once the initial transients decay, the output $\hat{e}$ follows the input $\dot{\theta}$ according to equation (4.1.6). The rate at which the initial transients decay depends upon the root locations of the observer.

Figure 4.1.4 illustrates the same solution as Figure 4.1.3 except that the initial values are equal.

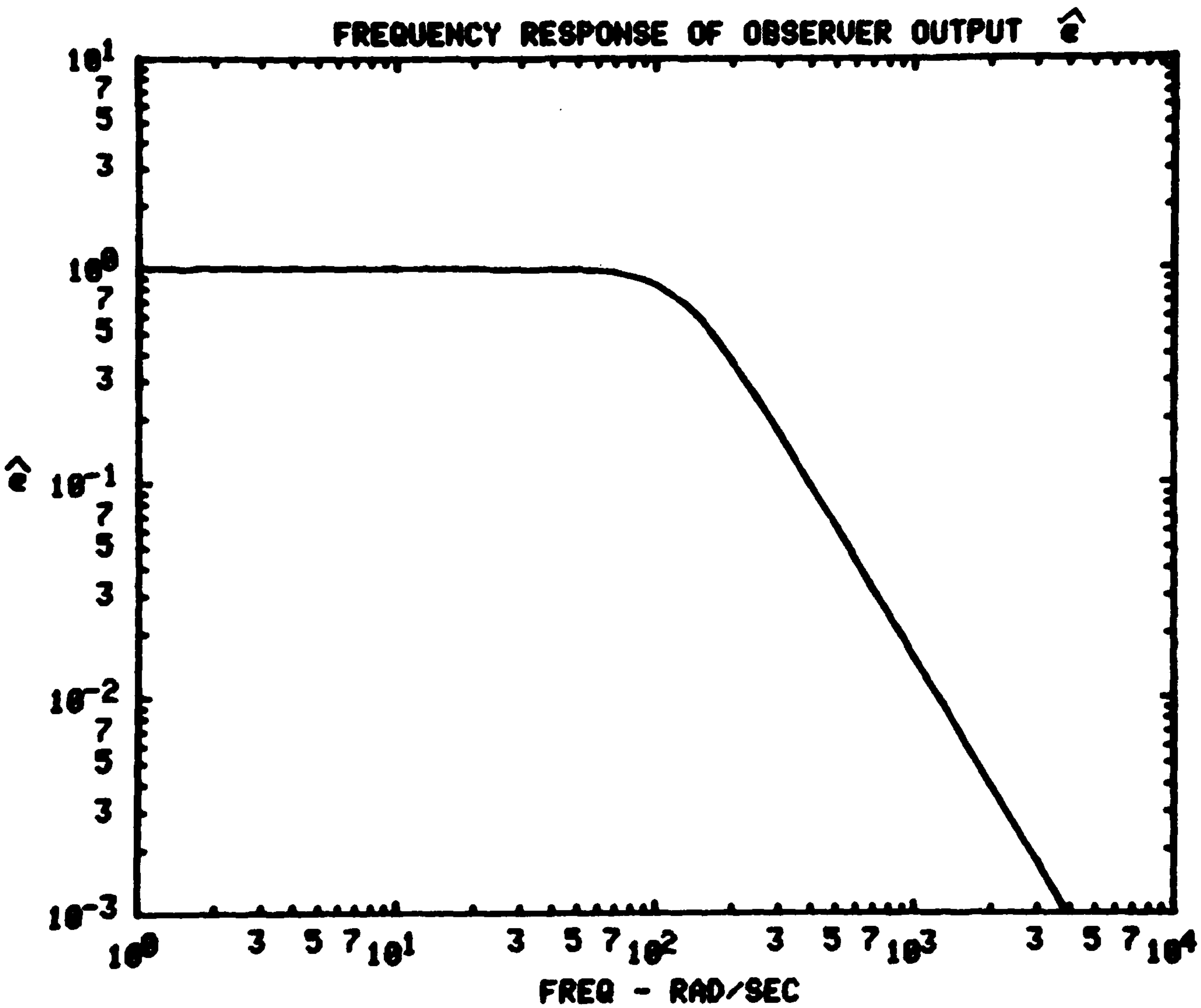
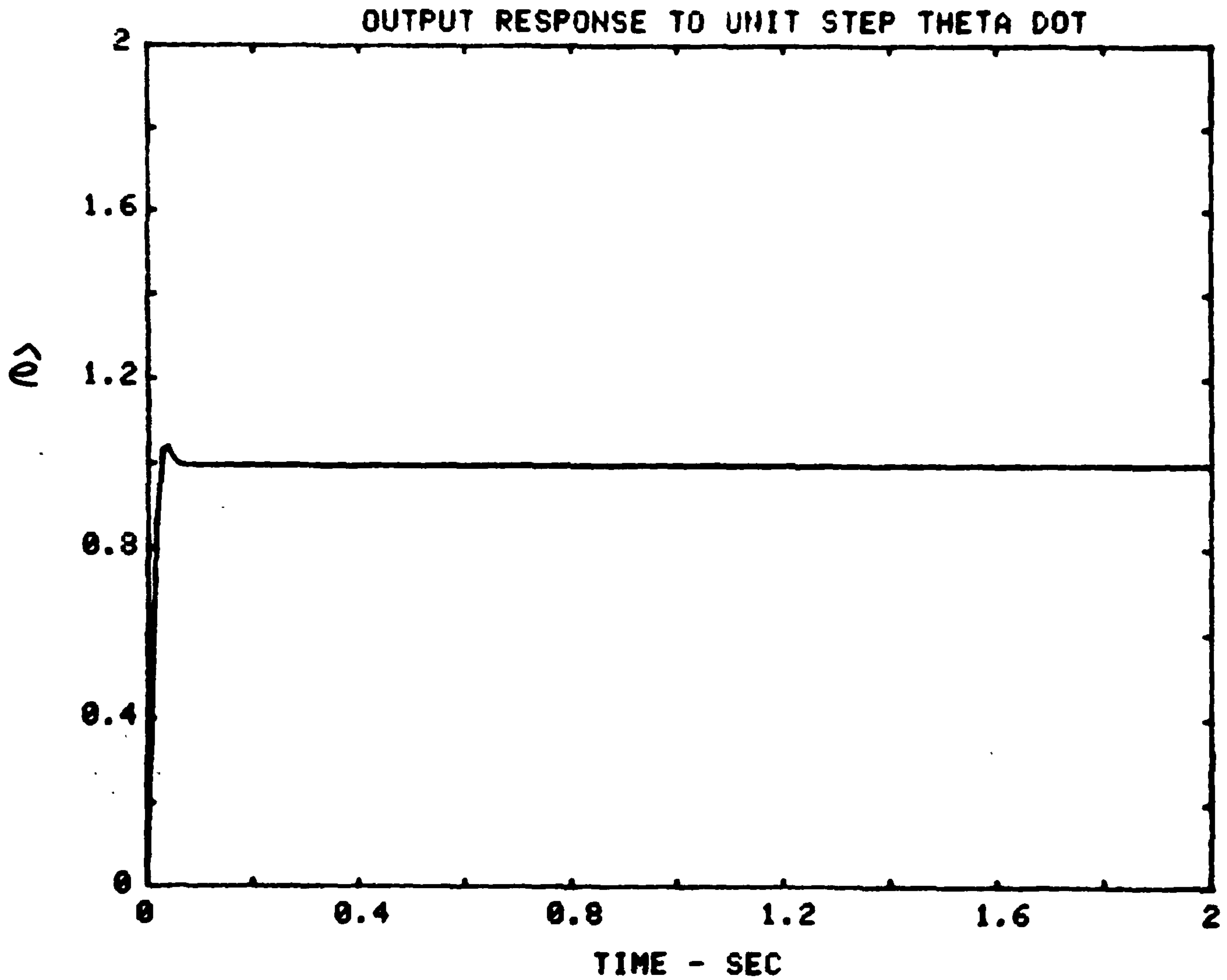

FREQUENCY RESPONSE OF OBSERVER OUTPUT $\hat{e}$
$\hat{e}$
10^{1} 7 5 3 10^{0} 7 5 3 10^{-1} 7 5 3 10^{-2} 7 5 3 10^{-3}
10^{0} 3 5 7 10^{1} 3 5 7 10^{2} 3 5 7 10^{3} 3 5 7 10^{4}
FREQ - RAD/SEC

FIGURE 4.1.1



OUTPUT RESPONSE TO UNIT STEP THETA DOT
2
1.6
1.2
0.8
0.4
0
0
0.4
0.8
1.2
1.6
2
TIME - SEC

FIGURE 4.1.2

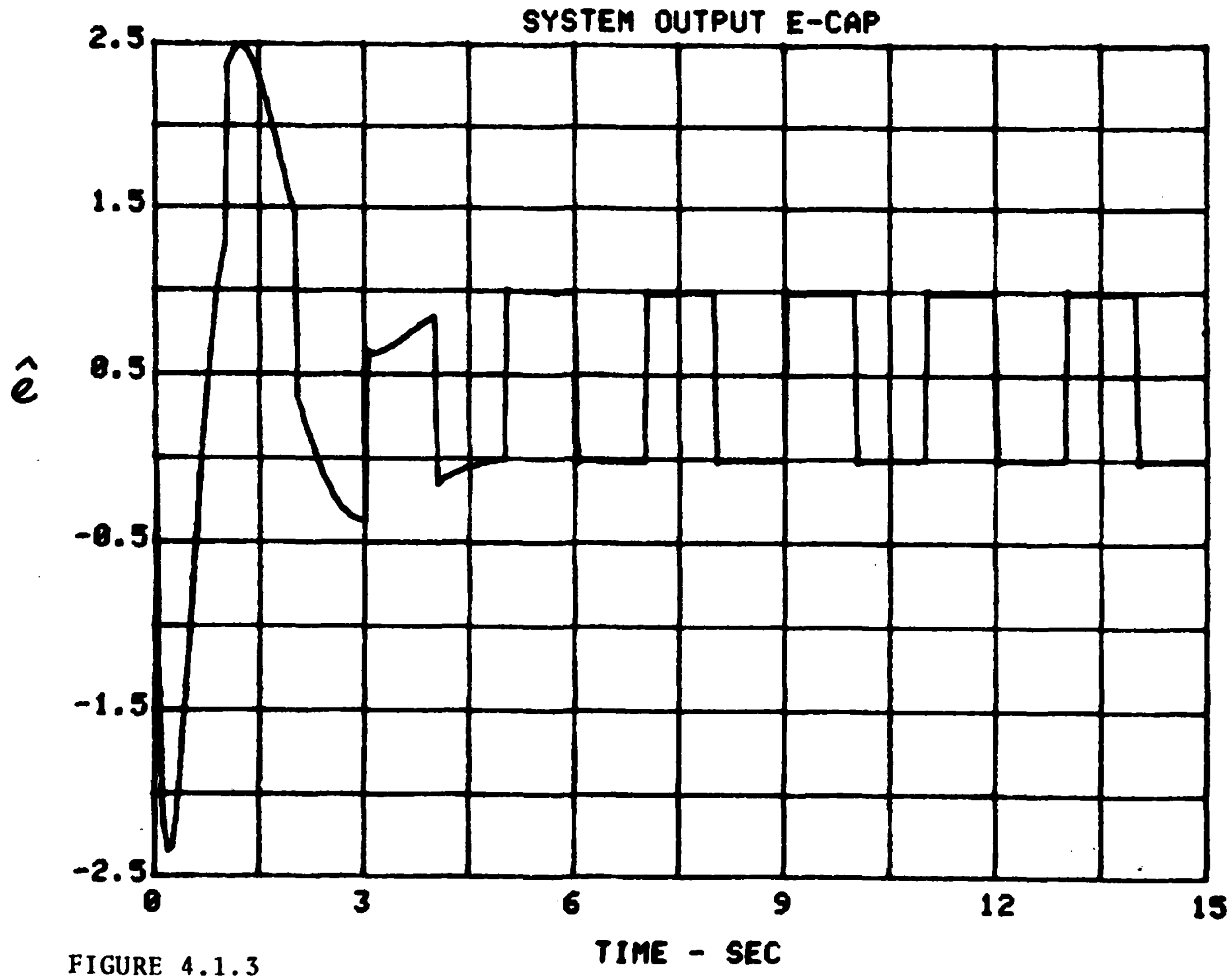


FIGURE 4.1.3

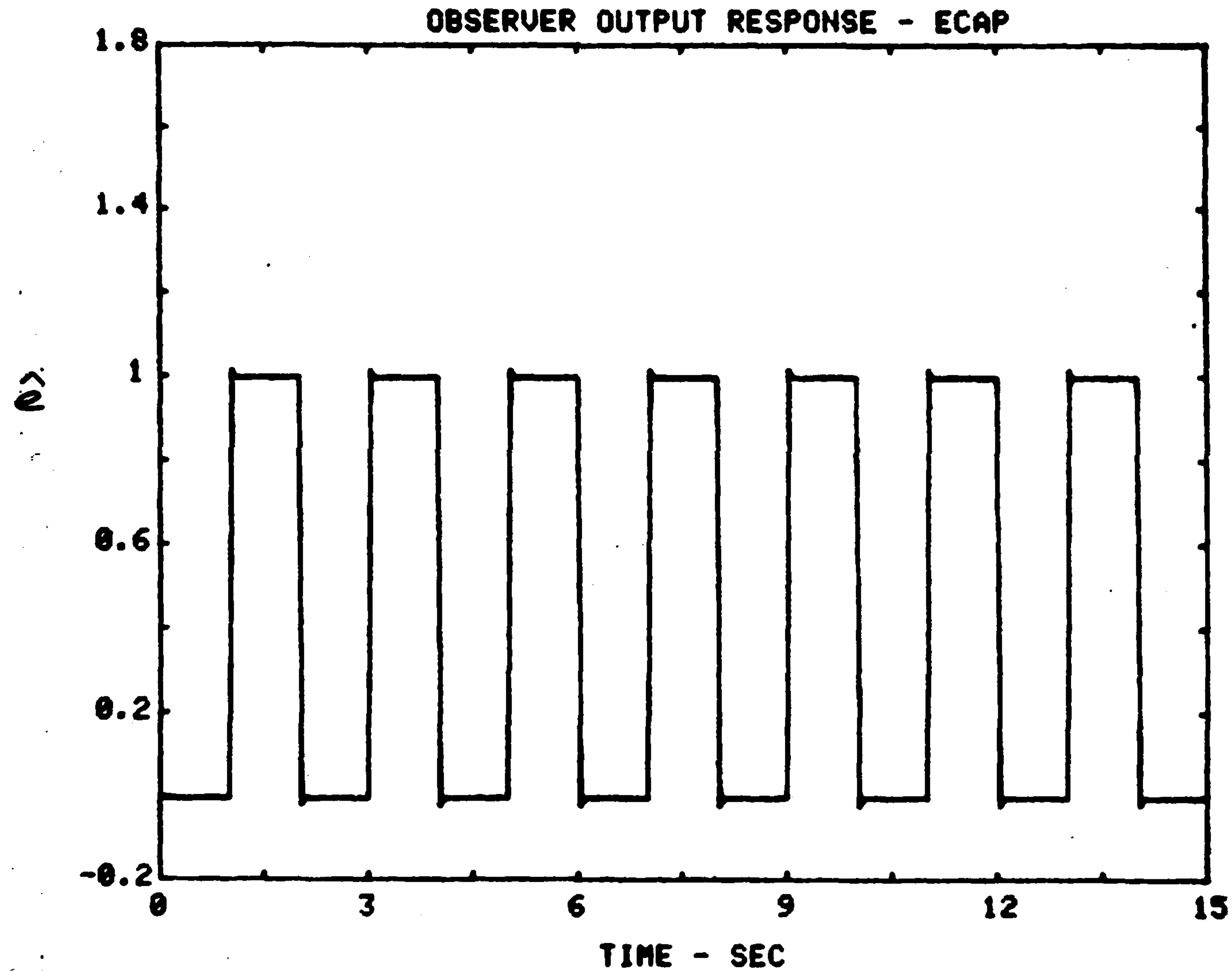

OBSERVER OUTPUT RESPONSE - ECAP
1.8
1.4
1
0.6
0.2
-0.2
0
3
6
9
12
15
TIME - SEC

FIGURE 4.1.4

5.0 OUTPUT SENSITIVITY WITH RESPECT TO THE OUTPUT MATRIX

5.1 Sensitivity Equations

The dynamics of the rate sensor are shown in Figure 2.2.1 as

$$e_o = C_4(sI-A)^{-1}B\theta \tag{5.1.1}$$

and the observer equations are

$$\hat{e} = M(sI-F)^{-1}(B\theta+\underline{K}e_o) \tag{5.1.2}$$

where M is the output matrix and is

$$M = [0 \quad m_2 \quad m_3 \quad m_4] \tag{5.1.3}$$

Physically, M is the summation of the observer states and was contrived to produce the desired output, thus the m_i's are the gains of a summing amplifier. Since

$$\mu^{\hat{e}}_{m_i} = m_i \frac{\partial \hat{e}}{\partial_{m_i}} \tag{5.1.4}$$

from equation (5.1.2) one may find that

$$\frac{\partial \hat{e}}{\partial m_i} = E^i_{1,4}(sI-F)^{-1}(B\theta+\underline{K}e_o) \tag{5.1.5}$$

Using (5.1.1), (5.1.5) becomes

$$\frac{\partial \hat{e}}{\partial m_i} = E^i_{1,4}(sI-F)^{-1}[I+\underline{K}C_4(sI-A)^{-1}]B\theta \tag{5.1.6}$$

which with the use of the theorem in (2.3.13) becomes

$$\frac{\partial \hat{e}}{\partial m_i} = E^i_{1,4}(sI-A)^{-1}B\theta \tag{5.1.7}$$

The $(sI-A)^{-1}$ and B matrix are known from (2.3.6) and (2.1.5) respectively, and so equation (5.1.7) becomes

$$\frac{\partial \hat{e}}{\partial m_i} = \frac{a}{A} E^{i}_{1,4} \begin{bmatrix} 1 \\ s \\ s^2 \\ s^3 \end{bmatrix} \theta \tag{5.1.8}$$

Now using equation (5.1.4) gives

$$\mu^{\hat{e}}_{m_2} = \frac{am_2}{\Delta_A} \dot{\theta} \tag{5.1.9}$$

$$\mu^{\hat{e}}_{m_3} = \frac{sam_3}{\Delta_A} \dot{\theta} \tag{5.1.10}$$

$$\mu^{\hat{e}}_{m_4} = \frac{s^2 am_4}{\Delta_A} \dot{\theta} \tag{5.1.11}$$

where Δ_A is the characteristic equation of the rate sensor.

If $\dot{\theta}$ is a unit step input, then under steady state conditions

$$\mu^{\hat{e}}_{m_2}\Big|_{ss} = \frac{am_2}{b} = 1 \tag{5.1.12}$$

and

$$\mu^{\hat{e}}_{m_3}\Big|_{ss} = \mu^{\hat{e}}_{m_4}\Big|_{ss} = 0$$

Therefore, deviations in the summer gain m_2 are directly reflected to the output $\hat{e}$. If $\dot{\theta}$ is a unit ramp, implying acceleration, then

$$\mu_{m_2}^{\hat{e}}\Big|_{ss} = \frac{1}{s} \qquad (5.1.13)$$

$$\mu_{m_3}^{\hat{e}}\Big|_{ss} = 4.6$$

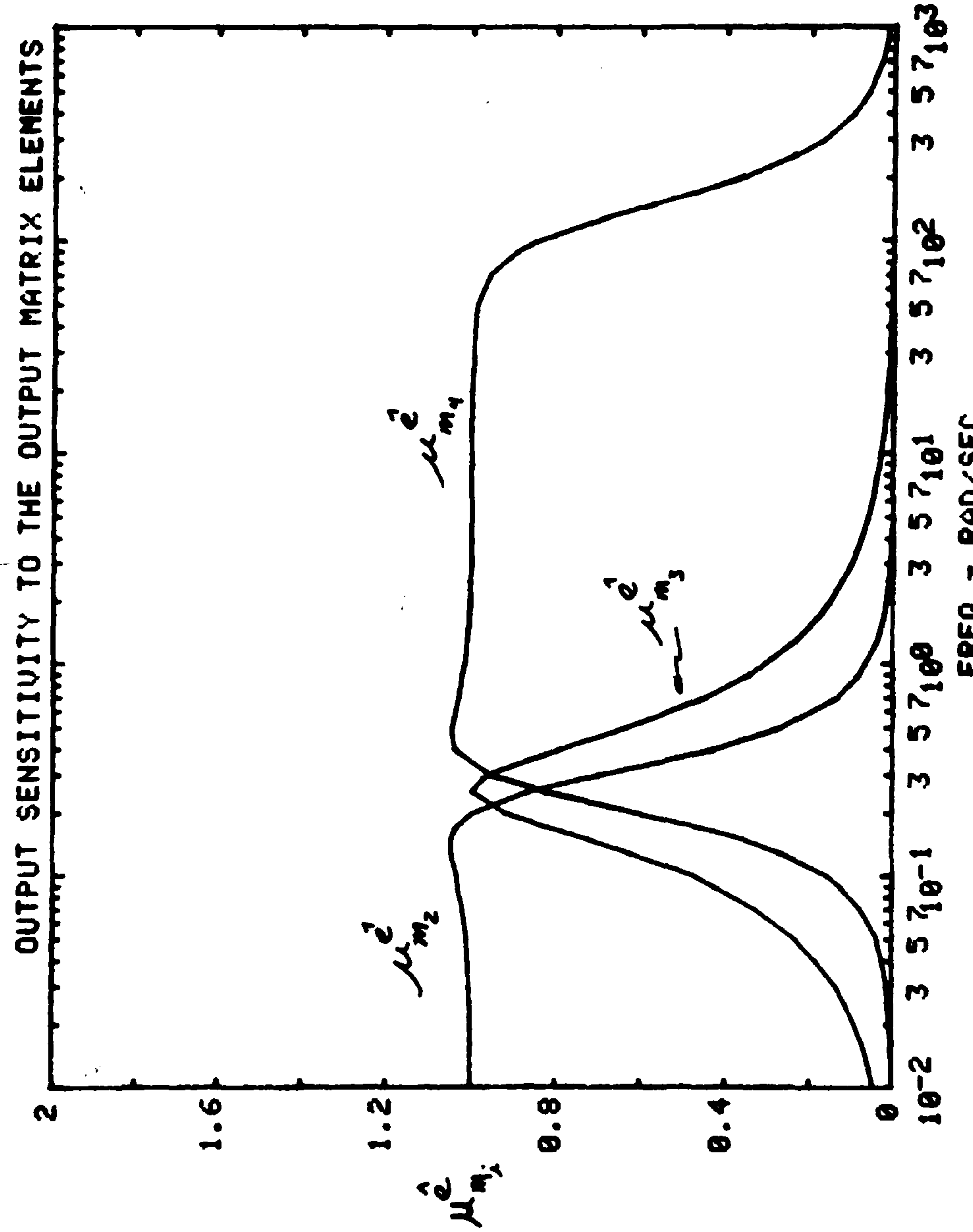
$$\mu_{m_4}^{\hat{e}}\Big|_{ss} = 0$$

According to this, at steady state, with $\dot{\theta}$ a ramp, deviations in $\hat{e}$ are 4.6 times the percent deviation in m_3 and the deviation in $\hat{e}$ grows with respect to errors in m_2. It would seem that this condition would seldom if ever exist for any length of time.

Figure 5.1.1 illustrates the frequency domain sensitivity of $\hat{e}$ to the m_i with a sine wave input at $\dot{\theta}$. The peak magnitudes are essentially the same, but the output $\hat{e}$ is sensitivity to each of the m_i over a different frequency range. As expected, only m_2 has an effect at zero frequency.

The curves of Figure 5.1.1 represent deviations in $\hat{e}$ for a 100% deviation in the m_i parameters. Thus, if m_2 is off by +10%, then the $\mu_{m_2}^{\hat{e}}$ curve of Figure 5.1.1 is multiplied by 0.1 to get $\Delta\hat{e}$. This is then added to the actual $\hat{e}$ response of Figure 4.1.1. Figures 5.1.2,3, and 4 illustrate the output response $\hat{e}$ to +10% deviations in m_2, m_3 and m_4 respectively.

The way the μ curve deviations add directly to the $\hat{e}$ curve is seen to be, $\hat{e}_{actual} = \hat{e}_{ideal} + \Delta\hat{e}$. Therefore, normally, only the deviation curves will be presented throughout the report.



OUTPUT SENSITIVITY TO THE OUTPUT MATRIX ELEMENTS
FREQ - RAD/SEC
2
1.6
1.2
0.8
0.4
0

FIGURE 5.1.1

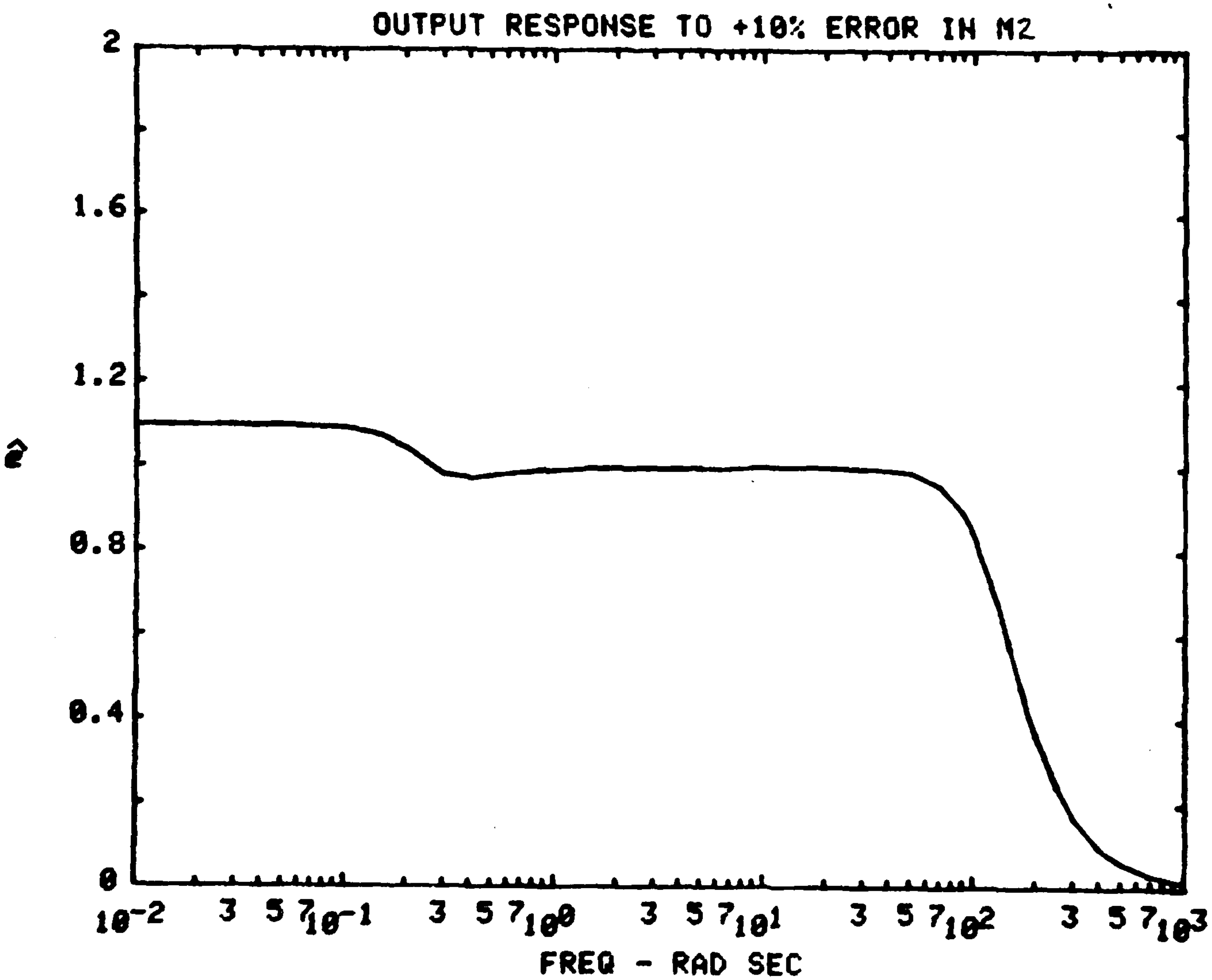

OUTPUT RESPONSE TO +10% ERROR IN M2
2
1.6
1.2
0.8
0.4
0
10-2 3 5 7 10-1 3 5 7 100 3 5 7 101 3 5 7 102 3 5 7 103
FREQ - RAD SEC

FIGURE 5.1.2

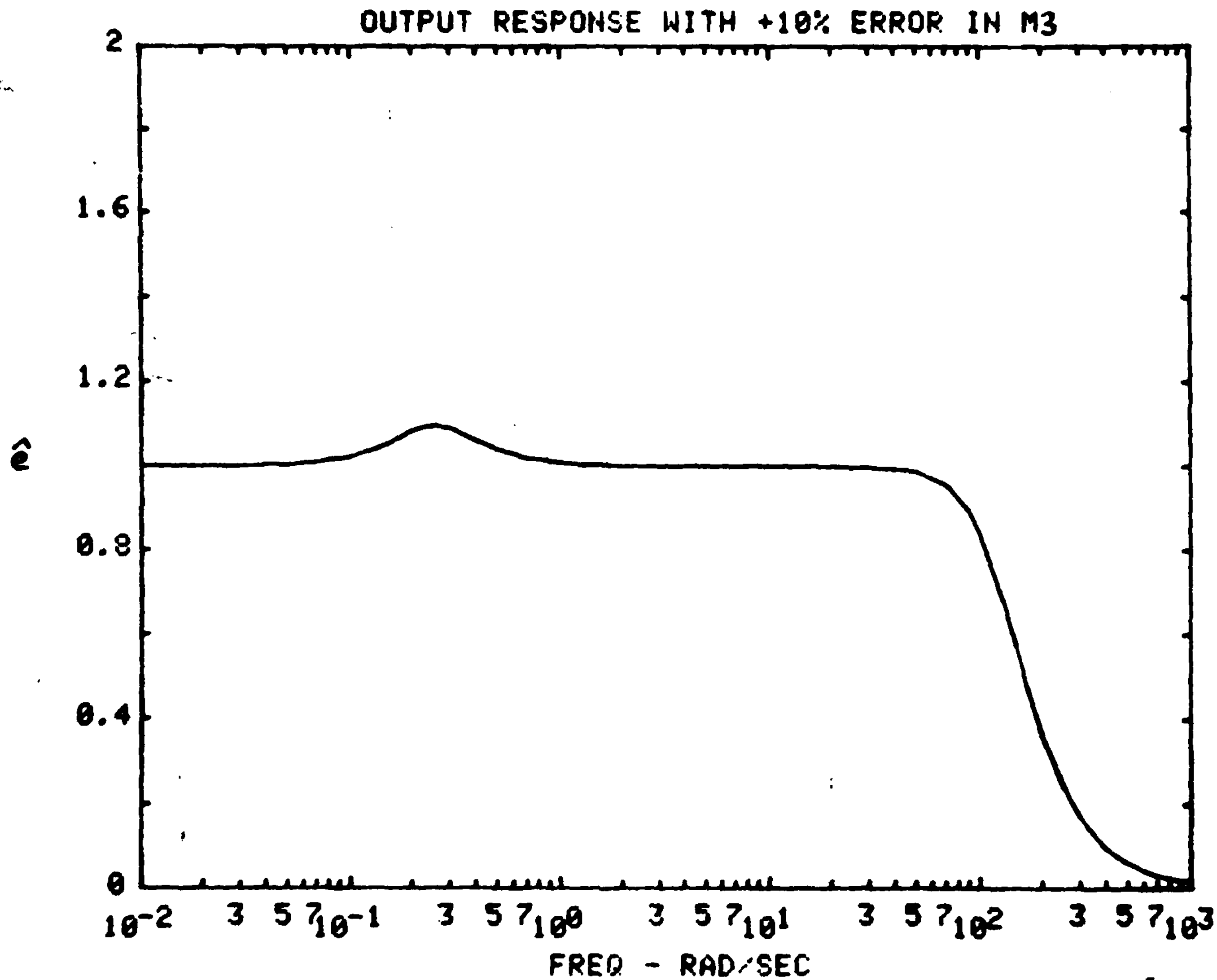

OUTPUT RESPONSE WITH +10% ERROR IN M3
2
1.6
1.2
0.8
0.4
0
ê
10-2 3 5 7 10-1 3 5 7 100 3 5 7 101 3 5 7 102 3 5 7 103
FREQ - RAD/SEC

FIGURE 5.1.3

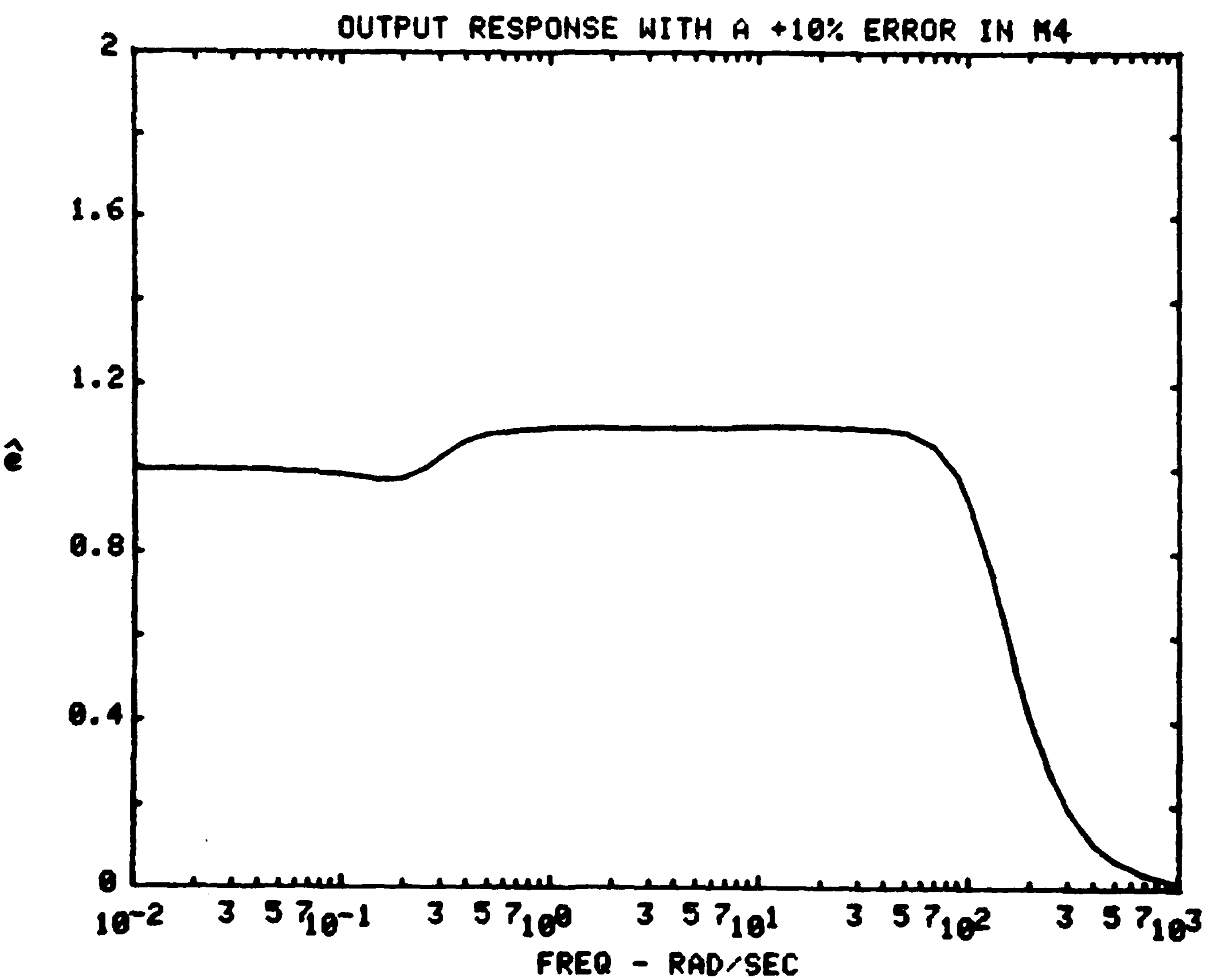


FIGURE 5.1.4

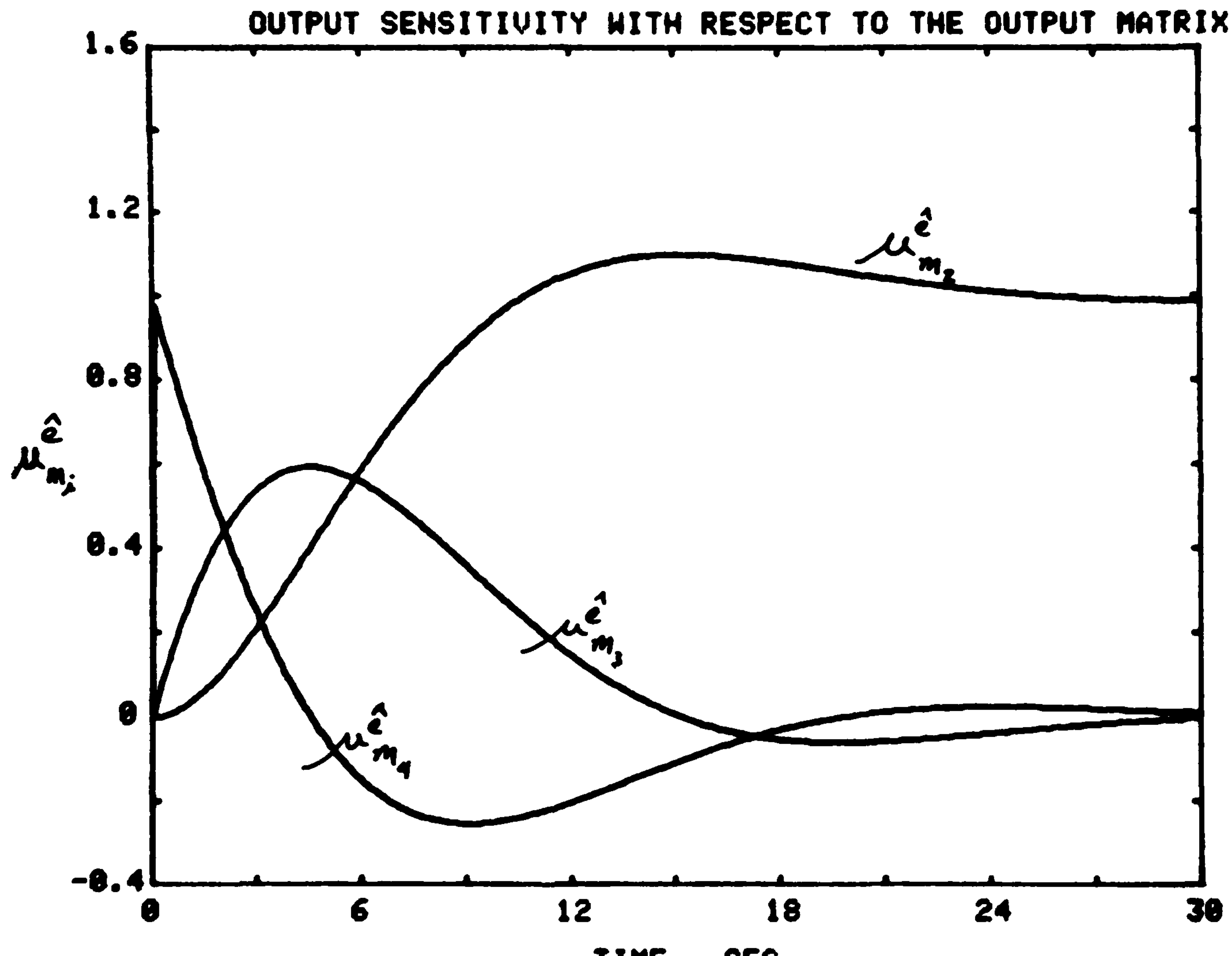
5.1.

Figure 5.1.5 illustrates the time response of the sensitivity equations (5.1.9-11) to a step input at $\dot{\theta}$. The differences in the frequency ranges are also apparent in these response curves. These curves, like the last ones, illustrate the deviation in the time response of the output, $\hat{e}$, to 100% deviations in the m_i parameters. Thus, if m_2 is in error by +10%, then the deviation in $\hat{e}$ is $\Delta\hat{e} = 0.1\mu^{\hat{e}}_{m_2}$. This response is added to the response of $\hat{e}$ with no deviations. That is, $\hat{e}_{actual} = \hat{e}_{ideal} + \Delta\hat{e}$. Figure 5.1.6 shows the output $\hat{e}$ with +10% error in m_2, m_3 and m_4, one at a time.

Again, if one takes 0.1μ from the sensitivity curves of Figure 5.1.5 and adds it to the e response curve of Figure 4.1.2, the result is Figure 5.1.6. The response of the system to the step input at $\dot{\theta}$ is very quick, but the errors in the m_i coefficients cause long term transients.

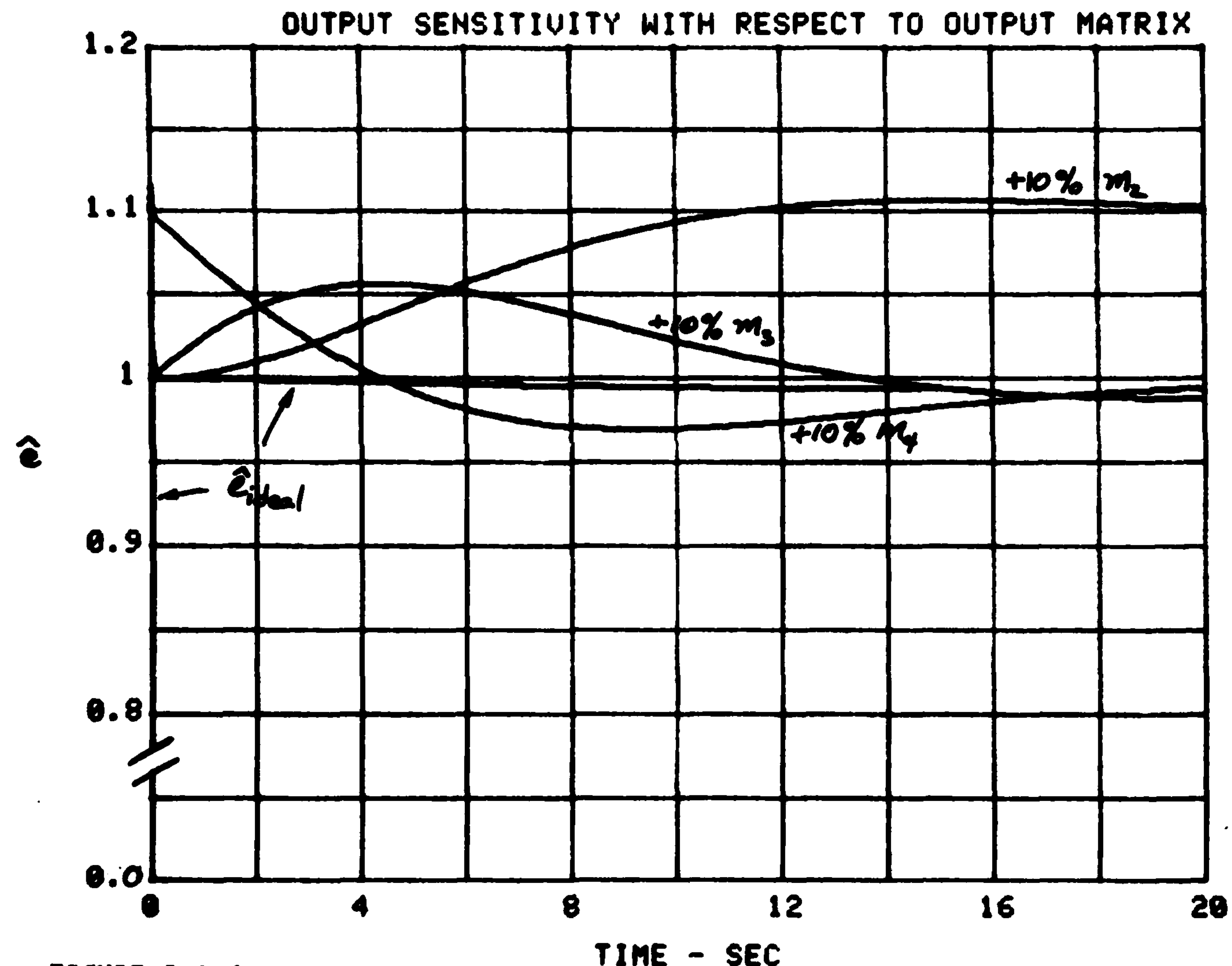
5.2 Results

Figure 5.2.1 illustrates a solution of the state equations for $\hat{e}$. Curve 1 is a response with +10% error in m_2, which implies that e should go to a steady state value of 1.1, since $\hat{e}_{actual} = (1+0.1\mu^{\hat{e}}_{m_2})\hat{e}_{ideal}$. Curve 2 represents a solution of $\hat{e}$ under ideal conditions except that $\dot{\theta}$ is a unit step at t = 0, then has another step at t = 5 superimposed. Curve 3 is the same as curve 2 only m_2 is in error by +10%. Note that errors in m_2 do not affect the tracking performance of the observer, once transients due to initial start up have decayed, they do alter the steady state value. The above is under the



OUTPUT SENSITIVITY WITH RESPECT TO THE OUTPUT MATRIX
1.6
1.2
0.8
0.4
0
-0.4
$\mu^{\hat{e}}_{m_i}$
$\mu^{\hat{e}}_{m_2}$
$\mu^{\hat{e}}_{m_3}$
$\mu^{\hat{e}}_{m_4}$
0
6
12
18
24
30
TIME - SEC

FIGURE 5.1.5



OUTPUT SENSITIVITY WITH RESPECT TO OUTPUT MATRIX
1.2
1.1
1
0.9
0.8
0.0
0
4
8
12
16
20
TIME - SEC
$\hat{e}$
+10% m_2
+10% m_3
+10% m_4
$\hat{e}_{ideal}$

FIGURE 5.1.6

conditions of a unit step at $\dot{\theta}$.

If $\dot{\theta}$ is a ramp then the situation of (5.1.13) is in existence. Figure 5.2.2 illustrates this condition. Curve 1 represents the ramp input $\dot{\theta} = 0.2t$ under ideal conditions. Curve 2 represents the same input with m_2 in +10% error. Clearly the error grows. Curve 3 represents ideal conditions with $\dot{\theta} = .2t$ until $t = 5$ at which time $\dot{\theta} = .2t+1$. Curve 4 represents the same input as 3 only m_2 is in error by +10%. Again, the error grows.

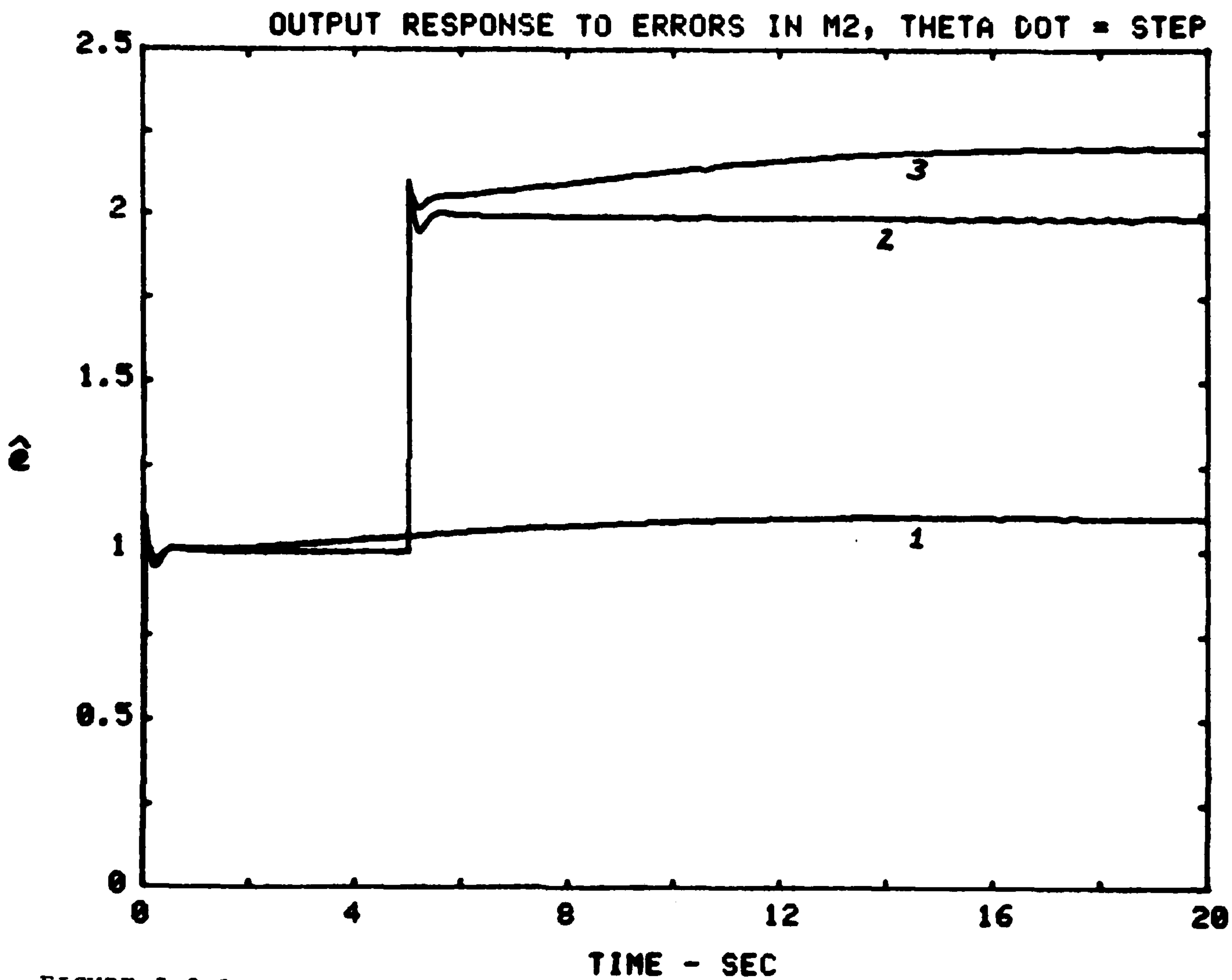
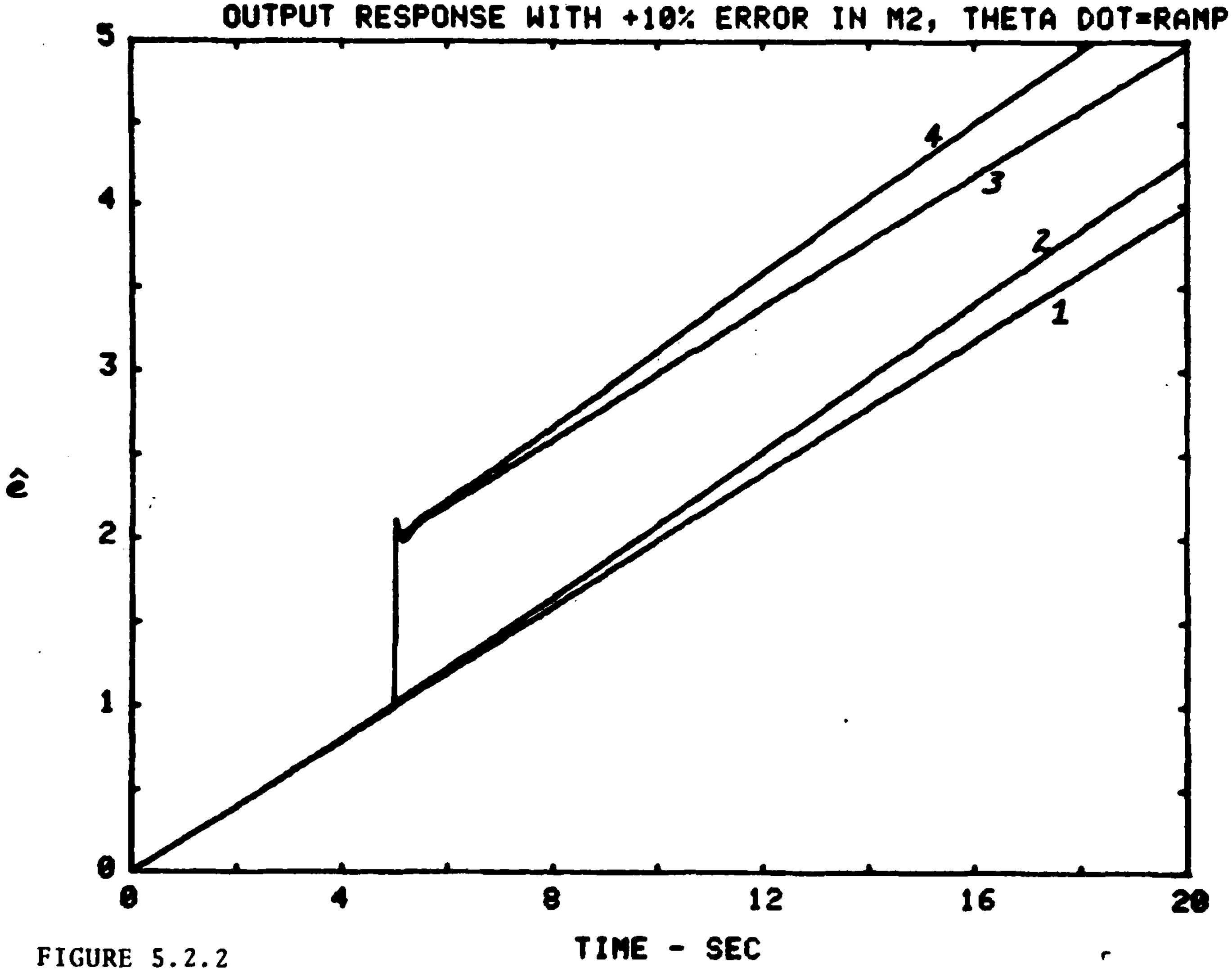

OUTPUT RESPONSE TO ERRORS IN M2, THETA DOT = STEP
2.5
2
1.5
1
0.5
0
ê
0
4
8
12
16
20
TIME - SEC
1
2
3

FIGURE 5.2.1



OUTPUT RESPONSE WITH +10% ERROR IN M2, THETA DOT=RAMP
ê
0
1
2
3
4
5
0
4
8
12
16
20
TIME - SEC
1
2
3
4

FIGURE 5.2.2

6.0 OUTPUT SENSITIVITY WITH RESPECT TO FORWARD PATH GAIN

6.1 Sensitivity Equations

Referring to Figure 2.2.1(b), the rate sensor equation is

$$e_o = C_4(sI-A)^{-1}B\theta \tag{6.1.1}$$

and the matrix B in (6.1.1) is

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ a \end{bmatrix}$$

where a is the forward path gain of the rate sensor. Also from Figure 2.2.1, the observer equation is

$$\hat{e} = M(sI-F)^{-1}(B\theta + \underline{K}e_o) \tag{6.1.2}$$

where the matrix B in (6.1.2) is the same as above, but in this case represents the forward path gain of the observer.

Under ideal conditions, these would be the same. However, if the gain of the rate sensor deviates from the ideal value, it affects $\hat{e}$ in a different manner than if the gain of the observer deviates.

The logarithmic sensitivity trajectory is

$$\mu_a^{\hat{e}} = a\frac{\partial \hat{e}}{\partial a} \tag{6.1.3}$$

in either case.

6.2 Sensitivity to Rate Sensor Gain

To find $\partial\hat{e}/\partial a$ of the rate sensor one starts with equation (6.1.2) and finds

$$\frac{\partial\hat{e}}{\partial a} = M(sI-F)^{-1}\underline{K}\,\frac{\partial\hat{e}_o}{\partial a} \tag{6.2.1}$$

and then from (6.1.1)

$$\frac{\partial e_o}{\partial a} = C_4(sI-A)^{-1}E^4_{4,1}\theta \tag{6.2.2}$$

where

$$E^4_{4,1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{6.2.3}$$

Therefore equation (6.2.1) becomes

$$\frac{\partial\hat{e}}{\partial a} = M(sI-F)^{-1}\underline{K}C_4(sI-A)^{-1}E^4_{4,1}\theta \tag{6.2.4}$$

If this is multiplied by a we get the sensitivity equation

$$\mu^{\hat{e}}_a = aM(sI-F)^{-1}\underline{K}C_4(sI-A)^{-1}E^4_{4,1}\theta \tag{6.2.5}$$

Using the theorem of (2.3.13), equation (6.2.5) may be written as

$$\mu^{\hat{e}}_a = aM[(sI-A)^{-1}-(sI-F)^{-1}]\,E^4_{4,1}\theta \tag{6.2.6}$$

The $(sI-A)^{-1}$ and $(sI-F)^{-1}$ matrices are given by (2.3.6) and (2.3.8) respectively. $E^4_{4,1}$ as given by (6.2.3) picks off the last column of each so that (6.2.6) becomes

$$\mu_a^{\hat{e}} = a[o \vdots m_2 m_3 m_4]\left(\frac{1}{\Delta_A}\begin{bmatrix} 1 \\ \hdashline s \\ s^2 \\ s^3 \end{bmatrix} - \frac{1}{\Delta_F}\begin{bmatrix} f_{14} \\ \hdashline f_{24} \\ f_{34} \\ f_{44} \end{bmatrix}\right)\theta \qquad (6.2.7)$$

where the f_{ij} are entries in the $(sI-F)^{-1}$ matrix.

Equation 6.2.7 reduces to

$$\mu_a^{\hat{e}} = \frac{a}{\Delta_F \Delta_{A2}} \left(m_4\Delta_F - m_4\Delta_A + \Delta_{A2}[\ m_3K_3 + m_2K_2)s + m_2K_3]\right)\dot{\theta} \qquad (6.2.8)$$

Figure 6.2.1 illustrates the sensitivity of $\hat{e}$ to the rate sensor gain in the frequency domain with a sine wave input at $\dot{\theta}$.

The output is quite sensitive to the sensor gain, however, the sensitivity decreases as the observer roots are made more negative, as discussed in Section 2.5, implying increased $\underline{K}$ gains.

If the right side of equation (6.2.2) is expanded the result is

$$\frac{\partial e_o}{\partial a} = \frac{s^3\theta}{\Delta_A} = \frac{s^2\dot{\theta}}{\Delta_A} \qquad (6.2.9)$$

For a step or a ramp (implies acceleration) at $\dot{\theta}$, the steady state value of $\partial e_o/\partial a = o$. Hence variations in the rate sensor gain have no effect on the steady state value of $\hat{e}$. This is seen to be the case in Figure 6.2.1.

Figure 6.2.2 illustrates the sensitivity function in the time domain and Figures 6.2.3-4 illustrate the solution of $\hat{e}$ with $\pm$ 10% deviation in the rate sensor gain.

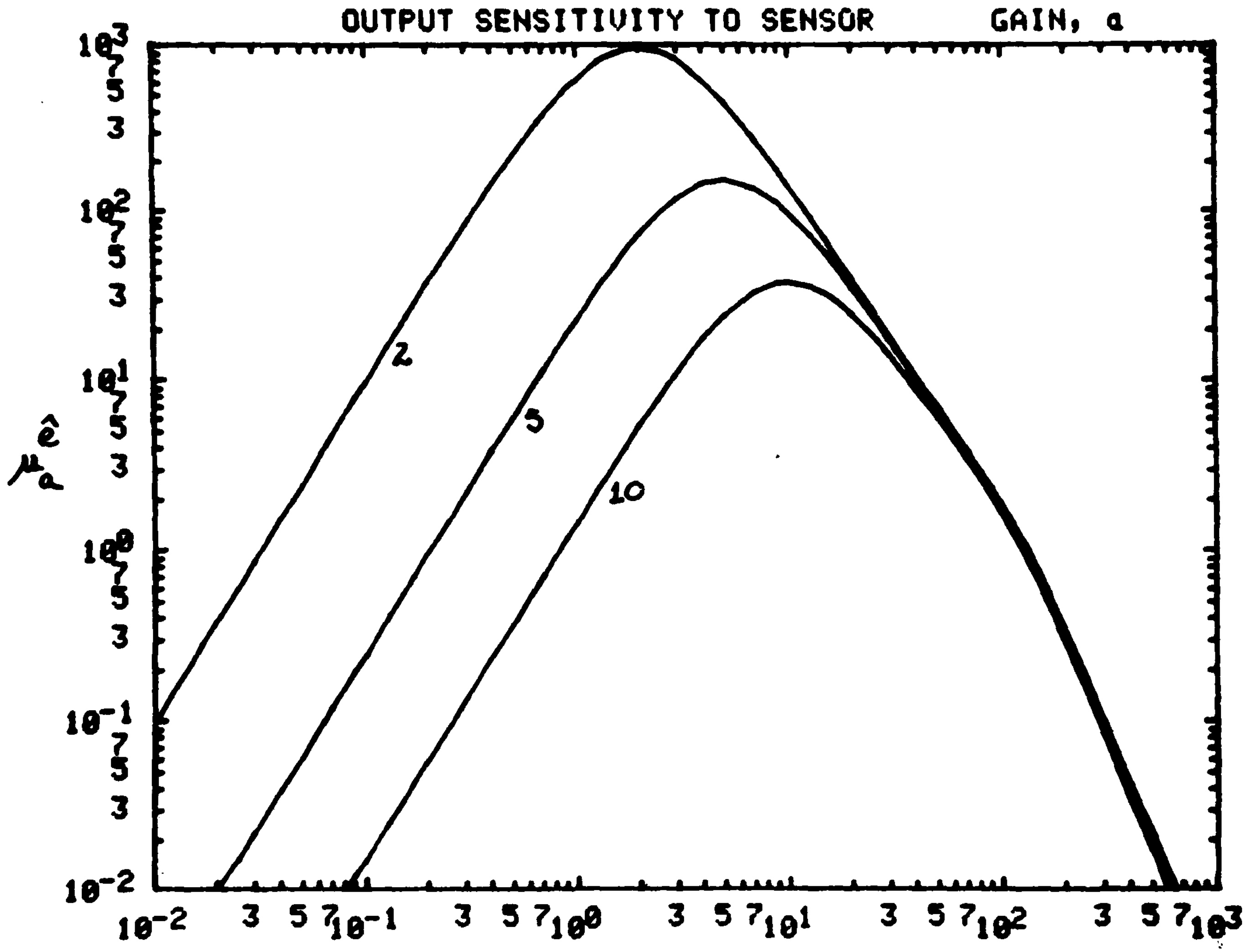
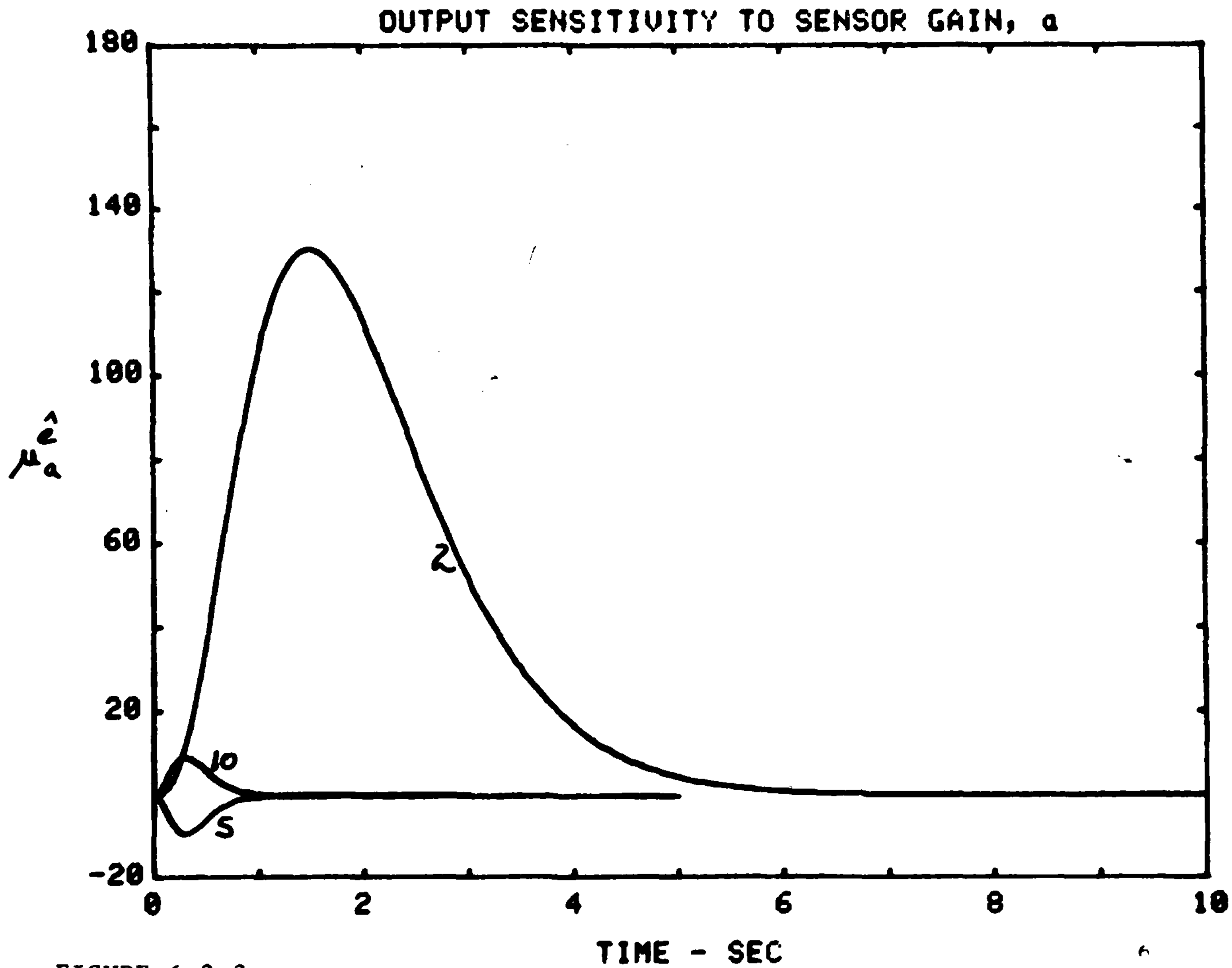

OUTPUT SENSITIVITY TO SENSOR GAIN, a
$\mu_a^{\hat{e}}$
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
2
5
10
10^{-2}
10^{-1}
10^0
10^1
10^2
10^3
FREQ - RAD/SEC

FIGURE 6.2.1



OUTPUT SENSITIVITY TO SENSOR GAIN, a
180
140
100
60
20
-20
$\hat{e}$
μ_a
0
2
4
6
8
10
TIME - SEC
2
10
5

FIGURE 6.2.2

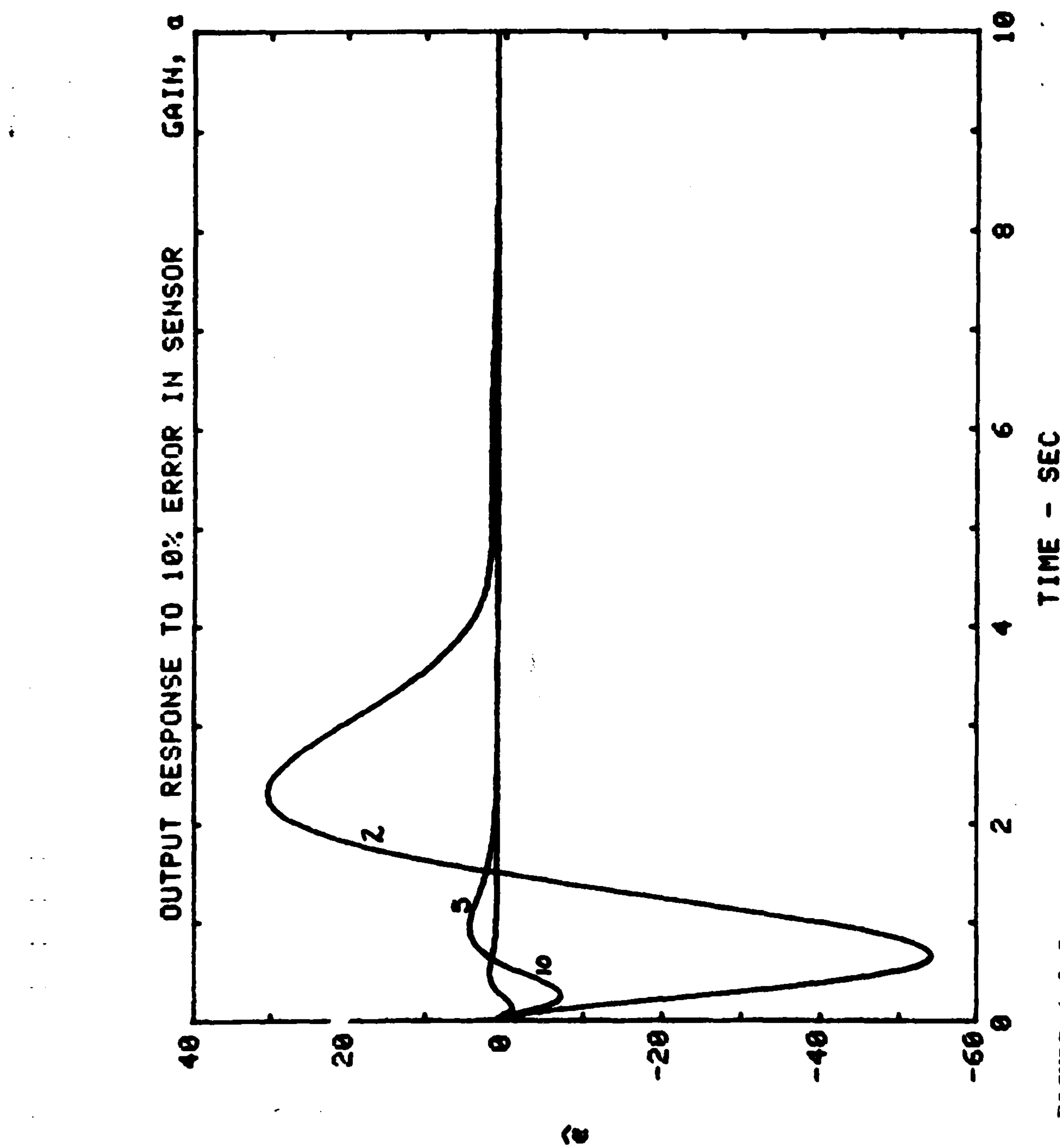

OUTPUT RESPONSE TO 10% ERROR IN SENSOR
GAIN, a
TIME - SEC
0
2
4
6
8
10
40
20
0
-20
-40
-60
2
5
10

FIGURE 6.2.3

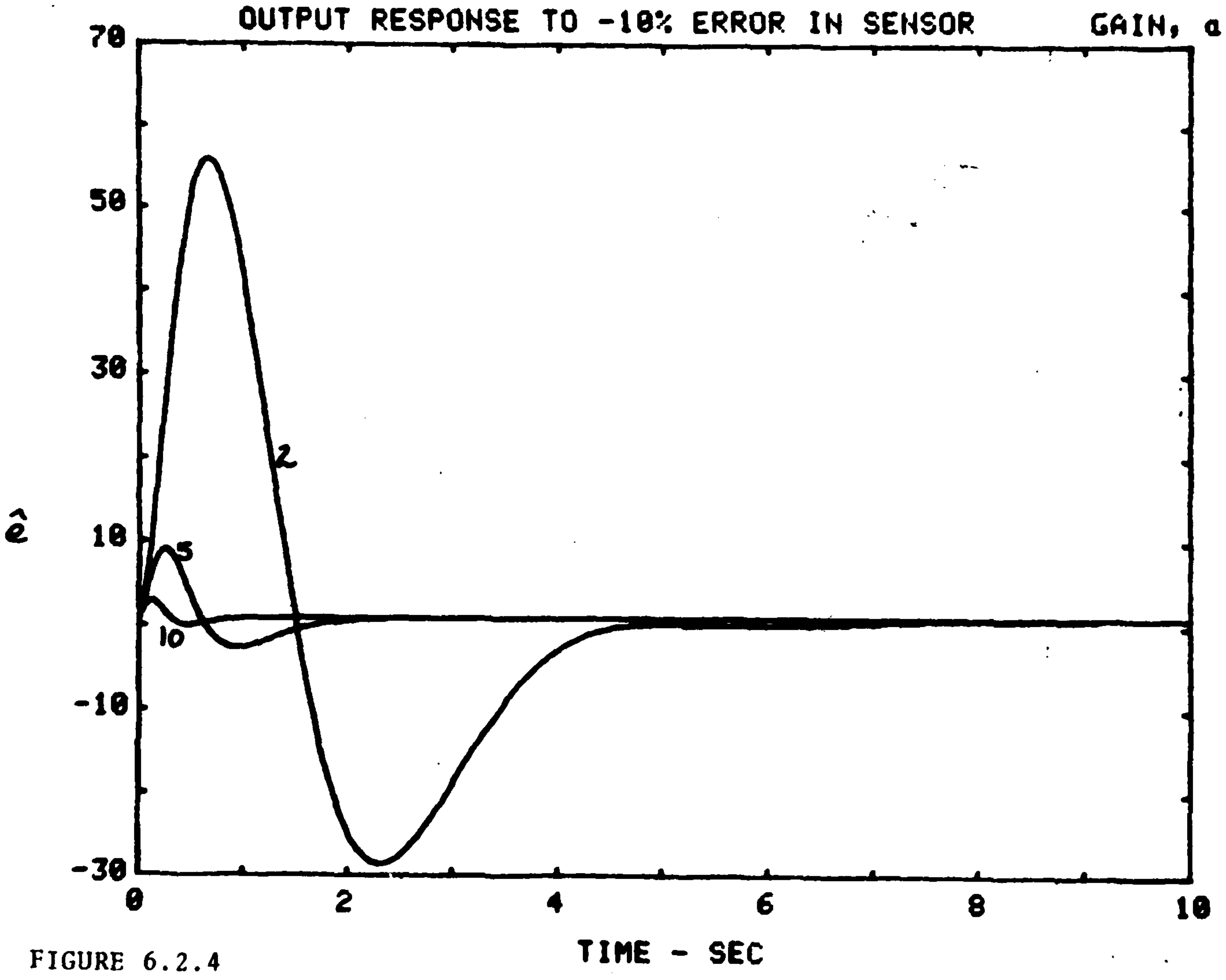

OUTPUT RESPONSE TO -10% ERROR IN SENSOR
GAIN, a
70
50
30
10
-10
-30
ê
0
2
4
6
8
10
TIME - SEC
2
5
10

FIGURE 6.2.4

Additional transients are introduced due to the mismatch of gains.

6.3 Sensitivity to Observer Gain

In this case, the gain a is the one in the B matrix of (6.1.2); hence from (6.1.2)

$$\frac{\partial \hat{e}}{\partial a} = M(sI-F)^{-1}E^4_{4,1}\ \theta \tag{6.3.1}$$

and so

$$\mu^{\hat{e}}_a = a\ M(sI-F)^{-1}E^4_{4,1}\theta \tag{6.3.2}$$

Matrix $E^4_{4,1}$ picks off the last column of $(sI-F)^{-1}$ from (2.3.8) resulting in

$$\mu^{\hat{e}}_a = \frac{a}{\Delta_f}\ [m_2 m_3 m_4] \begin{bmatrix} f_{24} \\ f_{34} \\ f_{44} \end{bmatrix} \theta \tag{6.3.3}$$

or

$$\mu^{\hat{e}}_a = \frac{a}{\Delta_f}\ [m_4\Delta_{A1} - (m_3K_3+m_2K_2)s - m_2K_3]\ \dot{\theta} \tag{6.3.4}$$

Figure 6.3.1 illustrates the frequency domain sensitivity of the output due to deviations in the observer forward path gain. Note that there is a steady state deviation in ê.

In equation (6.3.3), if $\dot{\theta}$ is a unit step, θ is a unit ramp and since

$$\begin{bmatrix} f_{24} \\ f_{34} \\ f_{44} \end{bmatrix} = \begin{bmatrix} -s^2K_2 - s(K_3-1) \\ -s^2(K_3-1) \\ s^3 \end{bmatrix}$$

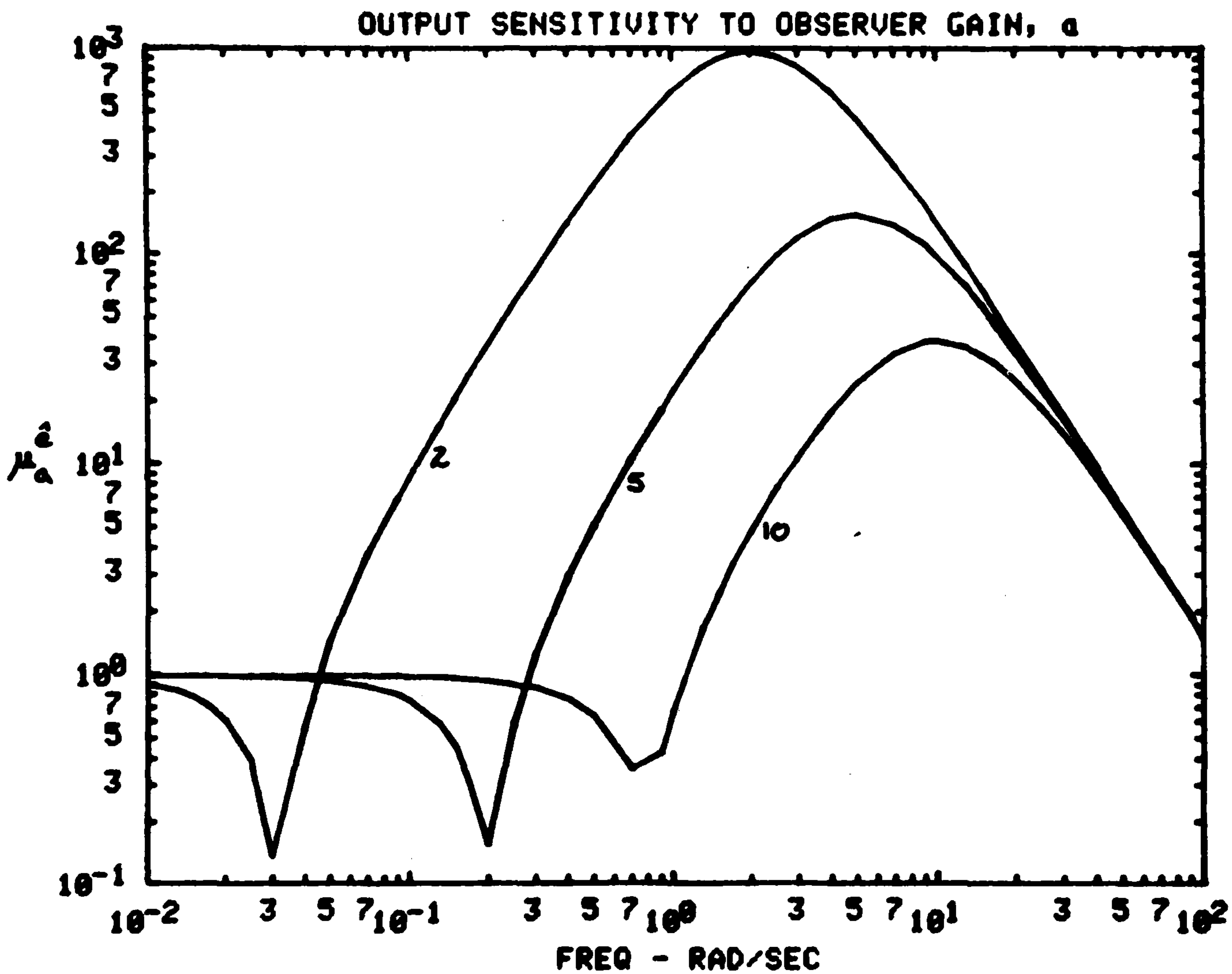

OUTPUT SENSITIVITY TO OBSERVER GAIN, a
$\mu_a^{\hat{e}}$
10^3
7
5
3
10^2
7
5
3
10^1
7
5
3
10^0
7
5
3
10^{-1}
10^{-2}
3
5
7
10^{-1}
3
5
7
10^0
3
5
7
10^1
3
5
7
10^2
2
5
10
FREQ - RAD/SEC

FIGURE 6.3.1

then the steady state value of   is

$$\mu_a^{\hat{e}} = \frac{am_2}{b} = 1 \tag{6.3.5}$$

and so

$$\Delta\hat{e}_{ss} = \frac{\Delta a}{a} \left.\mu_a^{\hat{e}}\right|_{ss} = \left.\frac{\Delta a}{a}\right|_{ss} \tag{6.3.6}$$

Thus if the observer roots are at -10, and if the observer gain is in error by 10%, the deviation is 0.1 of the -10 curve in Figure 6.3.1. That is, take 0.1 of the curve and add it to the ideal frequency response of Figure 4.1.1.

Figure 6.3.2 shows the time solution of the sensitivity equation (6.3.4) for a unit step input at $\mathring{\theta}$. Figures 6.3.3 and 6.3.4 illustrate the effect on the response of $\hat{e}$ with a $\pm$ 10% error in the observer gain a.

The final effect, or the steady state effect, of errors in the observer gain a is identical to the effect found for output matrix parameter m_2 in the last section. This is apparent in equation (6.3.5).

Figure 6.3.5 illustrates that the error in observer gain causes some additional transients, but the basic tracking ability still exists.

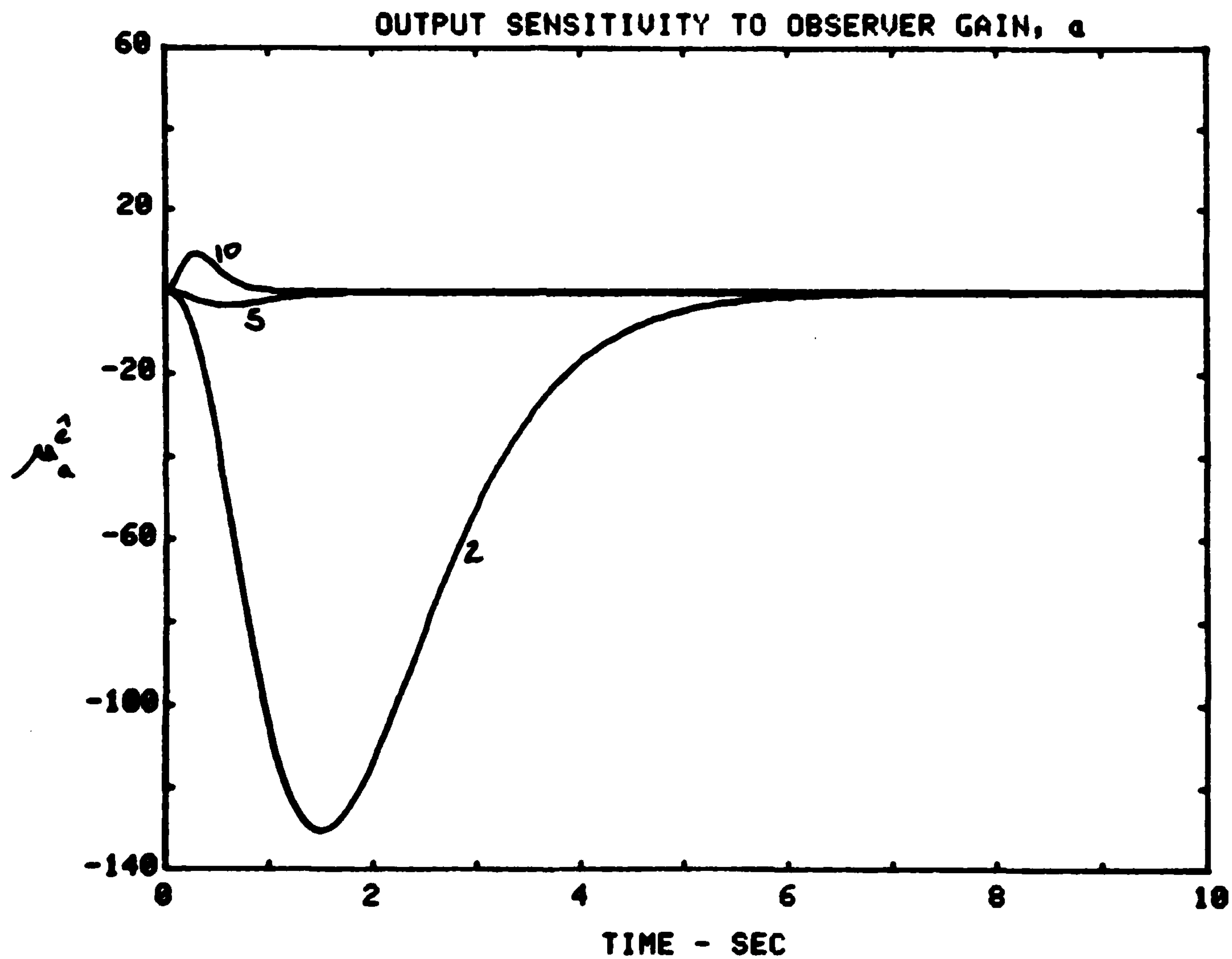

OUTPUT SENSITIVITY TO OBSERVER GAIN, a
60
20
-20
-60
-100
-140
0
2
4
6
8
10
TIME - SEC
10
5
2

FIGURE 6.3.2

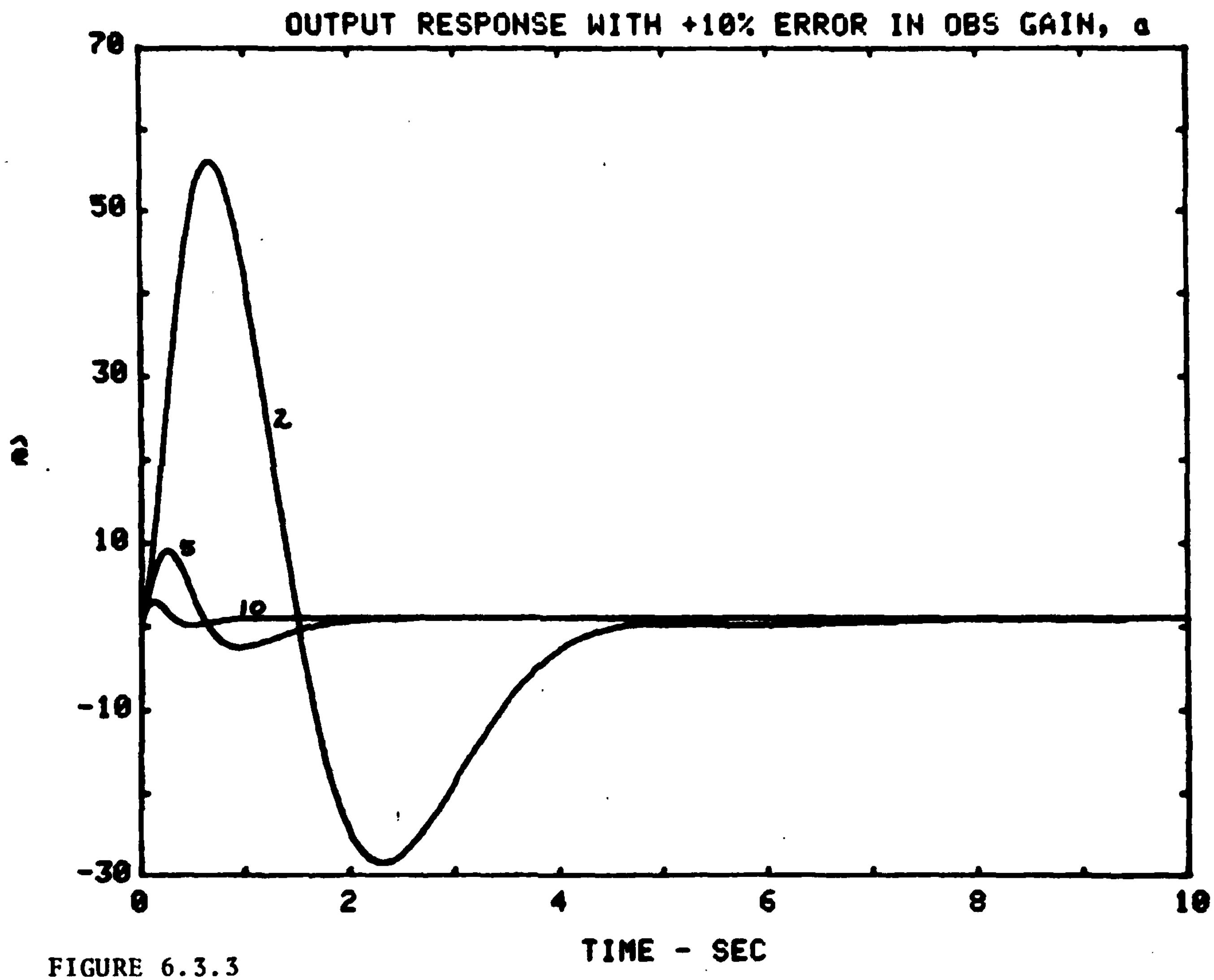

OUTPUT RESPONSE WITH +10% ERROR IN OBS GAIN, a
70
50
30
10
-10
-30
0
2
4
6
8
10
TIME - SEC
2
5
10

FIGURE 6.3.3

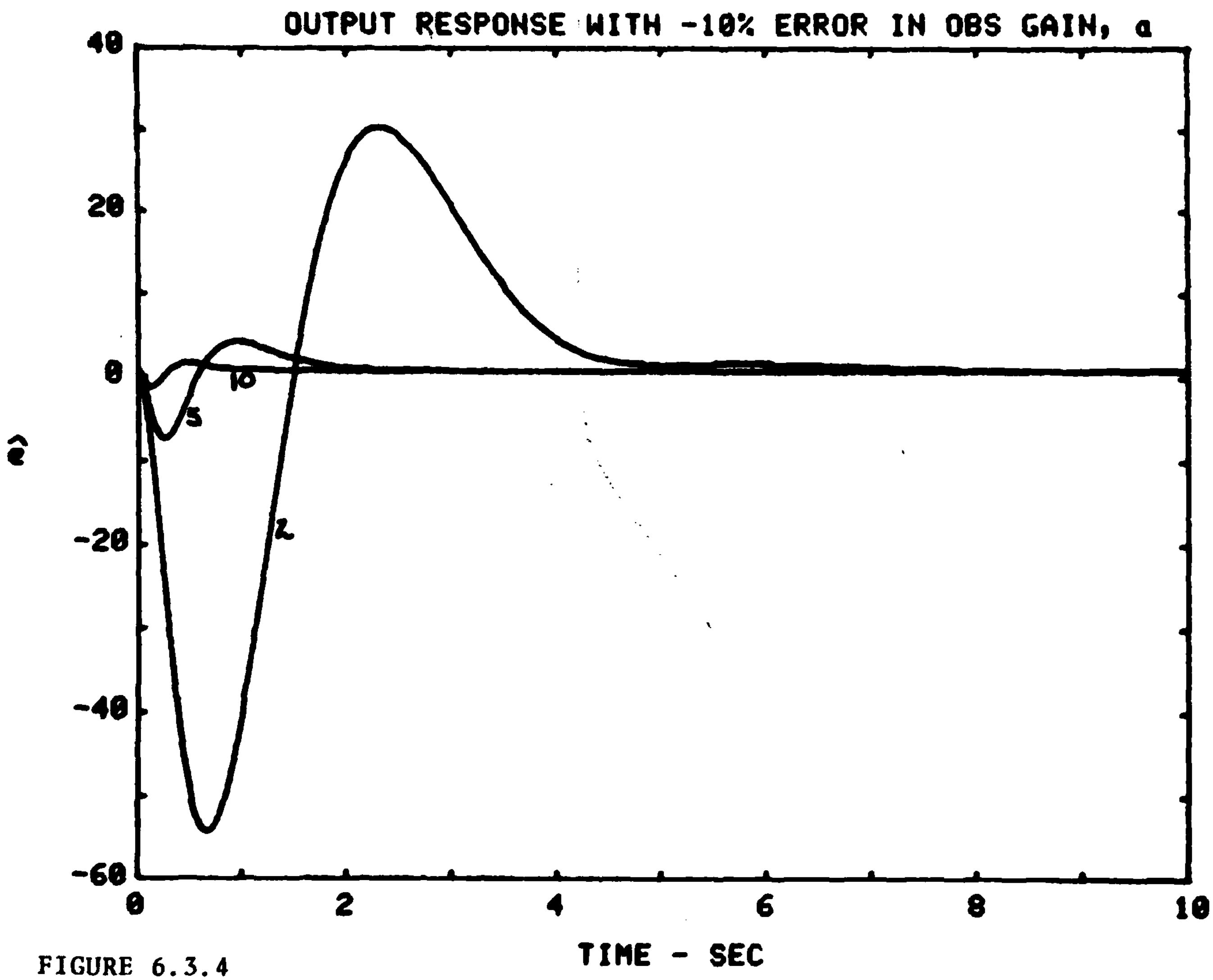
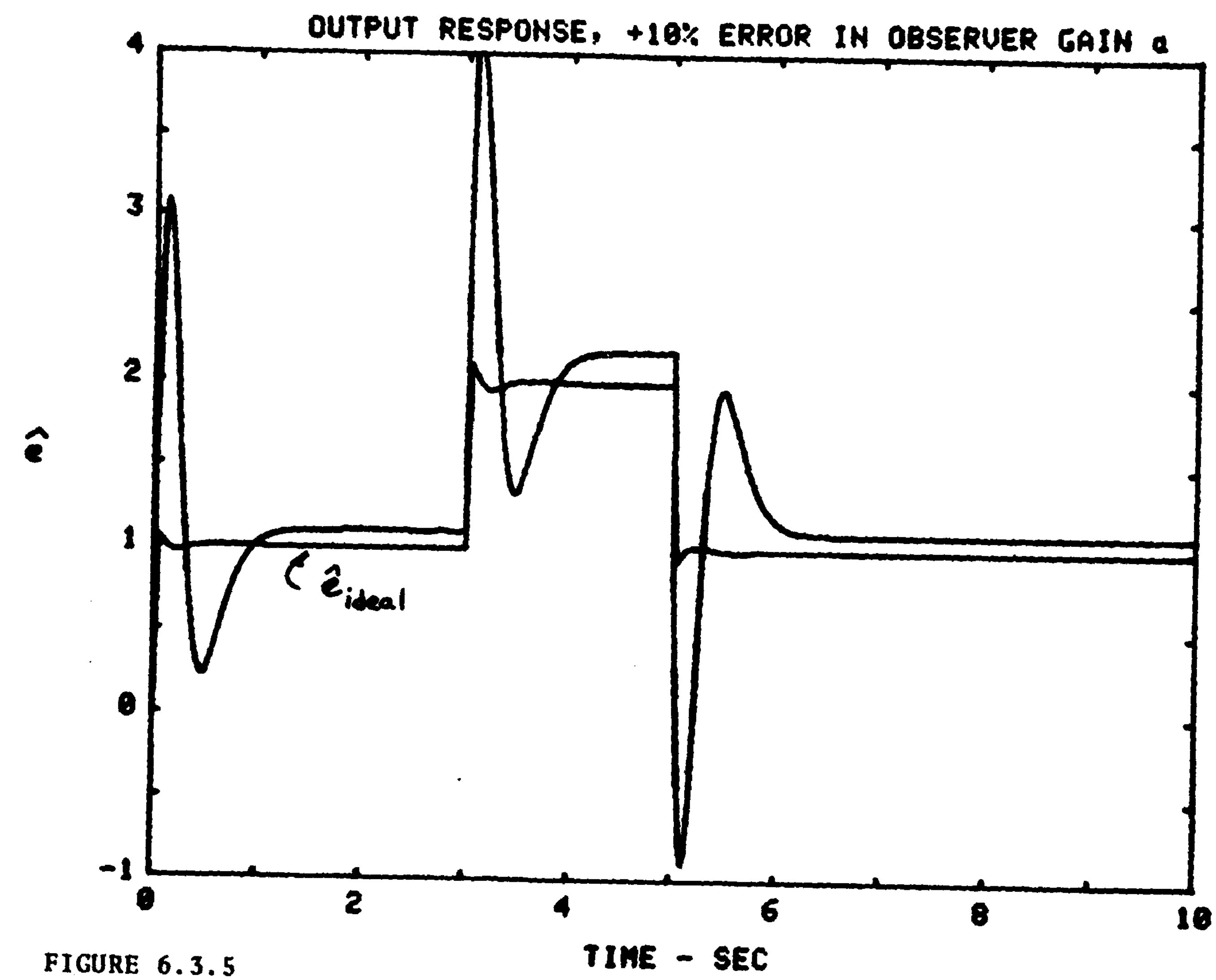

OUTPUT RESPONSE WITH -10% ERROR IN OBS GAIN, a
40
20
0
-20
-40
-60
ê
10
5
2
0
2
4
6
8
10
TIME - SEC

FIGURE 6.3.4



OUTPUT RESPONSE, +10% ERROR IN OBSERVER GAIN a
$\hat{e}$
4
3
2
1
0
-1
0
2
4
6
8
10
TIME - SEC
$\hat{e}_{ideal}$

FIGURE 6.3.5

7.0 OUTPUT SENSITIVITY WITH RESPECT TO THE OBSERVER GAINS

7.1 Sensitivity Equation

The rate sensor dynamics are defined by

$$e_o = C_4(sI-A)^{-1}B\theta \qquad (7.1.1)$$

and the observer dynamics are defined by

$$\hat{e} = M(sI-F)^{-1}(B\theta+\underline{K}e_o) \qquad (7.1.2)$$

and so the logarithmic sensitivity function desired is

$$\mu^{\hat{e}}_{K_i} = K_i\frac{\partial\hat{e}}{\partial K_i} \qquad (7.1.3)$$

From (7.1.2) one may write

$$\frac{\partial\hat{e}}{\partial K_i} = M(sI-F)^{-1}E^i_{4,1}e_o \qquad (7.1.4)$$

Combining (7.1.1) and (7.1.4) gives

$$\frac{\partial\hat{e}}{\partial K_i} = M(sI-F)^{-1}E^i_{4,1}\ C_4(sI-A)^{-1}B\theta \qquad (7.1.5)$$

Referring to equations (2.1.7) and (2.3.6), by inspection it is seen that the term $C_4(sI-A)^{-1}B$ reduces to as^3/Δ_A, so that

$$\frac{\partial\hat{e}}{\partial K_i} = \frac{as^2}{\Delta_A}\ M(sI-F)^{-1}E^i_{4,1}\dot{\theta} \qquad (7.1.6)$$

Equation 2.3.8 is the expression for $(sI-F)^{-1}$. Represent this as

$$(sI-F)^{-1} = \begin{bmatrix} f_{11} & f_{12} & f_{13} & f_{14} \\ f_{21} & f_{22} & f_{23} & f_{24} \\ f_{31} & f_{32} & f_{33} & f_{34} \\ f_{41} & f_{42} & f_{43} & f_{44} \end{bmatrix} \qquad (7.1.7)$$

and since $\mathbf{m} = [0\ m_2\ m_3\ m_4]$, equation (7.1.6) may be written as

$$\frac{\partial \hat{e}}{\partial K_i} = \frac{as^2}{\Delta_A \Delta_F} \sum_{\substack{j=2 \\ i=1}}^{4} (m_j f_{ji}) \mathring{\theta} \tag{7.1.8}$$

and then of course

$$\mu_{K_i}^{\hat{e}} = \frac{aK_i s^2}{\Delta_A \Delta_A} \sum_{\substack{j=2 \\ i=1}}^{4} (m_j f_{ji}) \mathring{\theta} \tag{7.1.9}$$

In all cases, even when $\mathring{\theta}$ is a ramp, the steady state sensitivity is zero.

Figures 7.1.1-4 illustrate the sensitivity of $\hat{e}$ to the various observer gains, $\underline{K}$. In general the system is more sensitive to gain K_3 because it is most closely tied to the output of the sensor. The deviation curves of Figures 7.1.1-4 may be added directly to the ideal response curve of Figure 4.1.1. Figures 7.1.5 and 7.1.6 illustrate this where the observer roots were all placed at -10 and then the observer gains, $\underline{K}$ were deviated by $\pm$ 10%, one at a time.

Figures 7.1.7-10 illustrate the time solution of the $\mu_{k_i}^{\hat{e}}$ equations in (7.1.9) to a unit step at $\mathring{\theta}$. Figures 7.1.11-14 illustrate the output response, $\hat{e}$ with a 10% error in the observer $\underline{K}$ gains. Figure 4.1.2 shows the response with no deviations.

Errors in the observer gains alter the observer output in the frequency band of approximately 0.1-100 rad/sec, depending upon which gain is considered. The errors cause no steady state deviation of $\hat{e}$.

OUTPUT SENSITIVITY TO OBSERVER GAIN K1

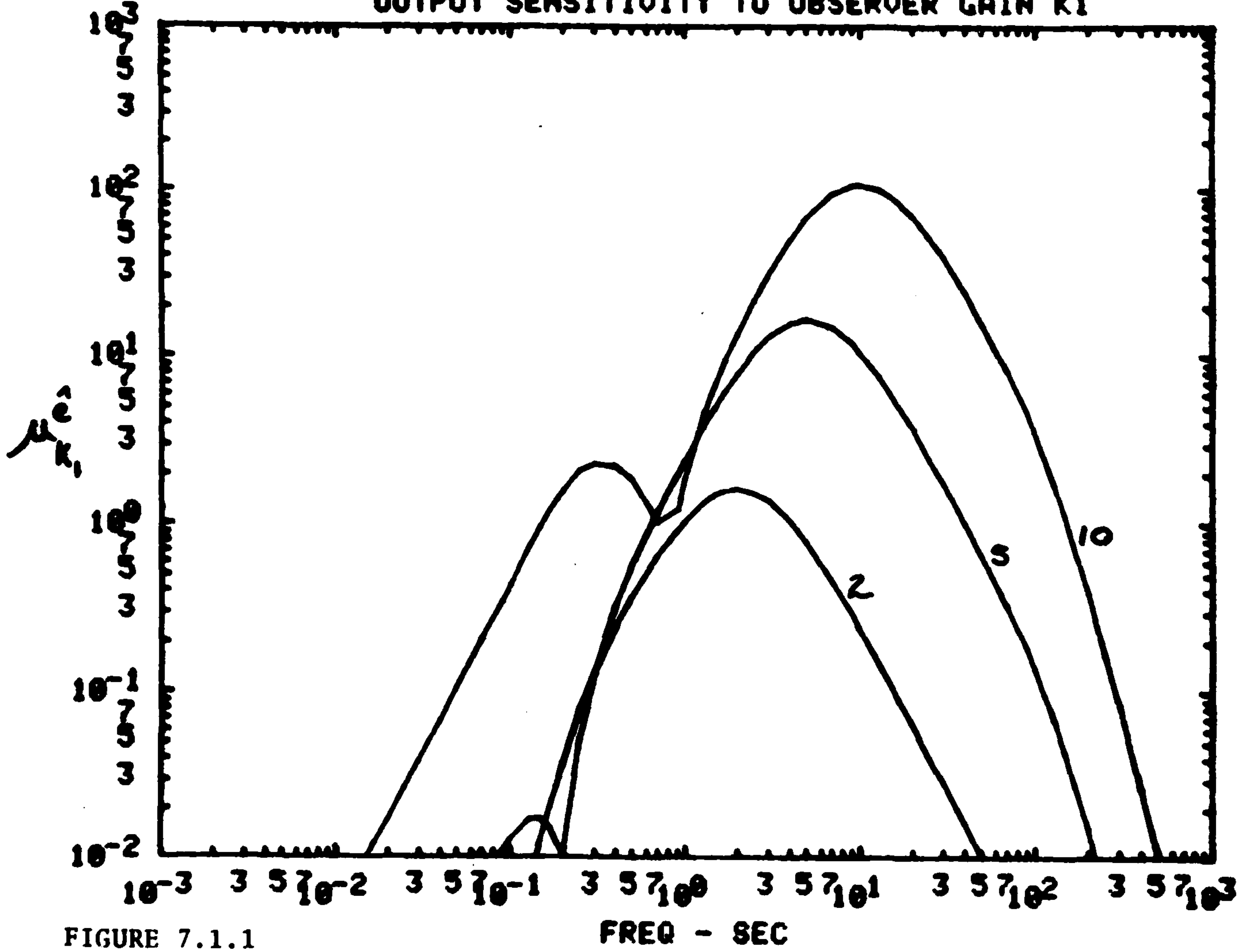
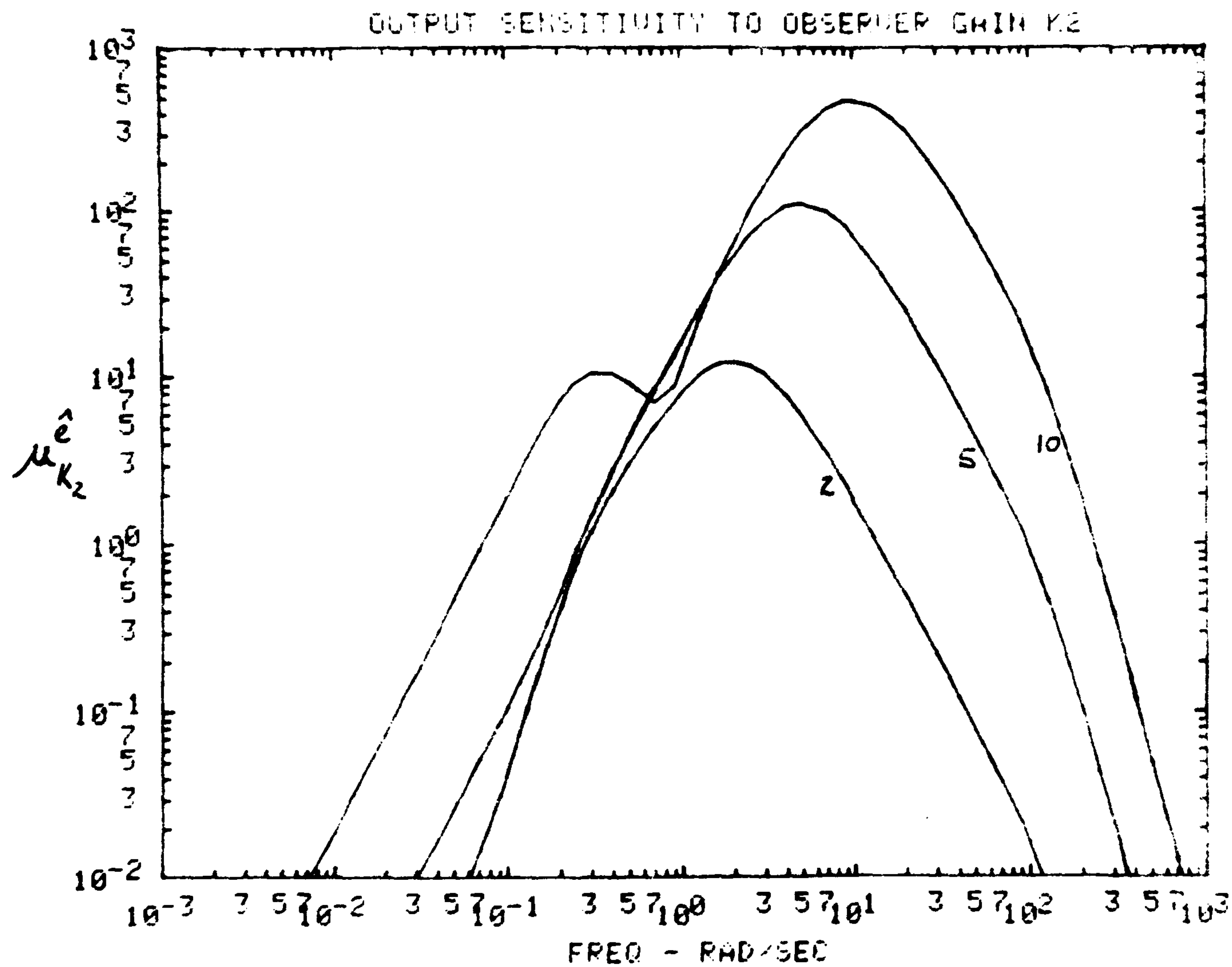


FIGURE 7.1.1


OUTPUT SENSITIVITY TO OBSERVER GAIN K2
$\mu^{\hat{e}}_{K_2}$
FREQ - RAD/SEC
2
5
10
FIGURE 7.1.2

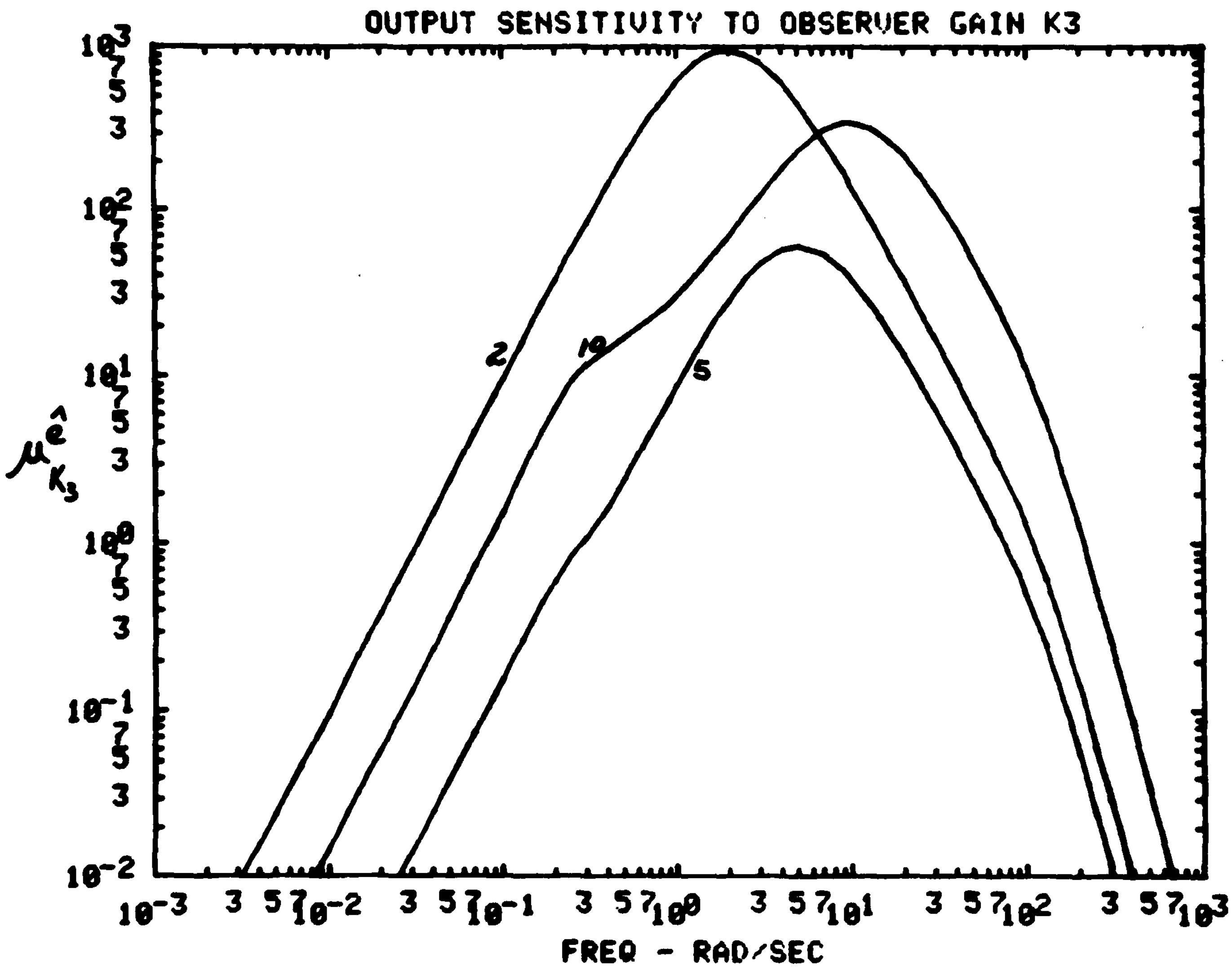
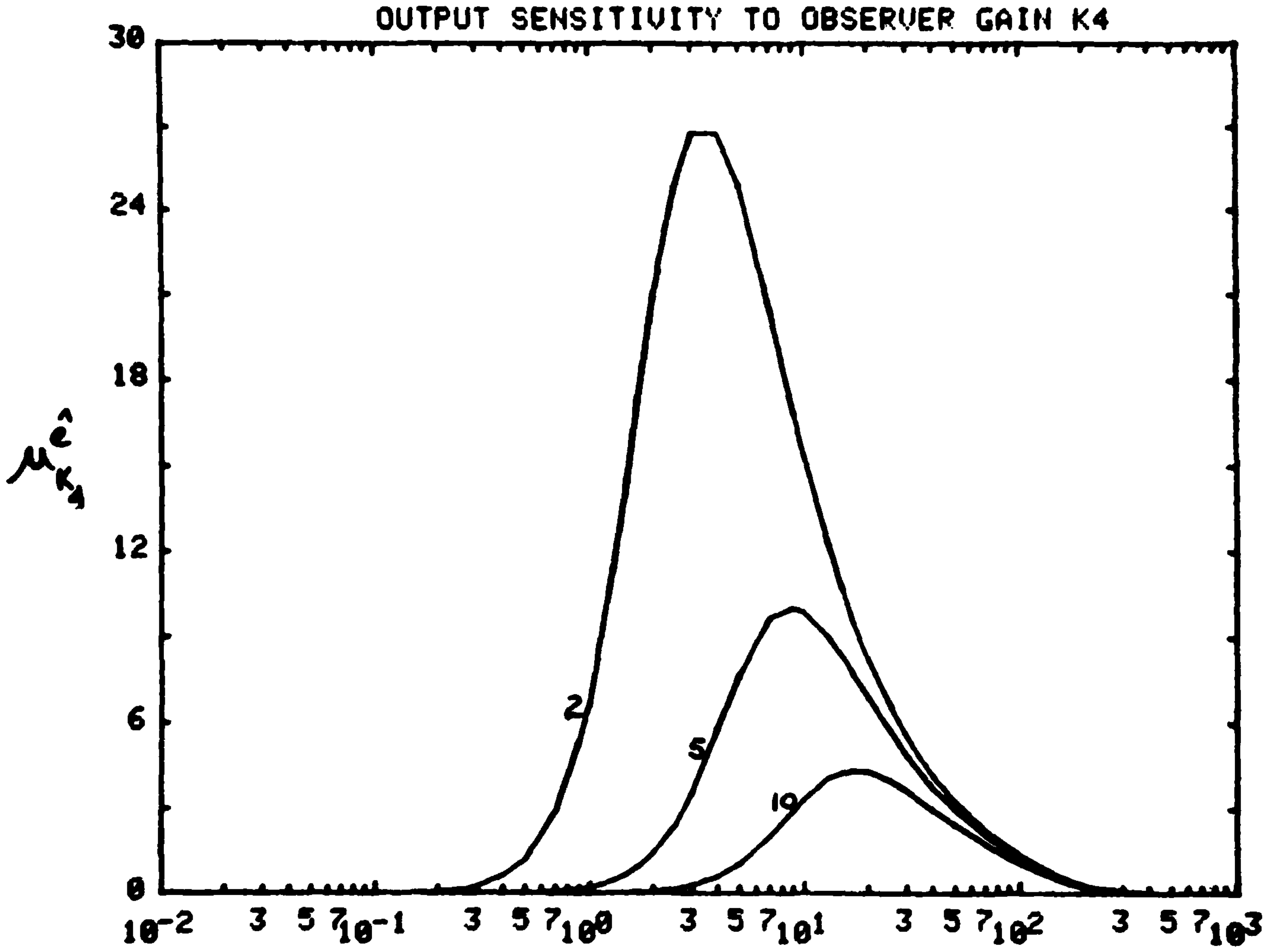

OUTPUT SENSITIVITY TO OBSERVER GAIN K3
$\hat{\mu}^{e}_{K_3}$
2
10
5
FREQ - RAD/SEC

FIGURE 7.1.3



OUTPUT SENSITIVITY TO OBSERVER GAIN K4
30
24
18
12
6
0
2
5
10
10-2 3 5 7 10-1 3 5 7 10⁰ 3 5 7 10¹ 3 5 7 10² 3 5 7 10³
FREQ - RAD/SEC

FIGURE 7.1.4

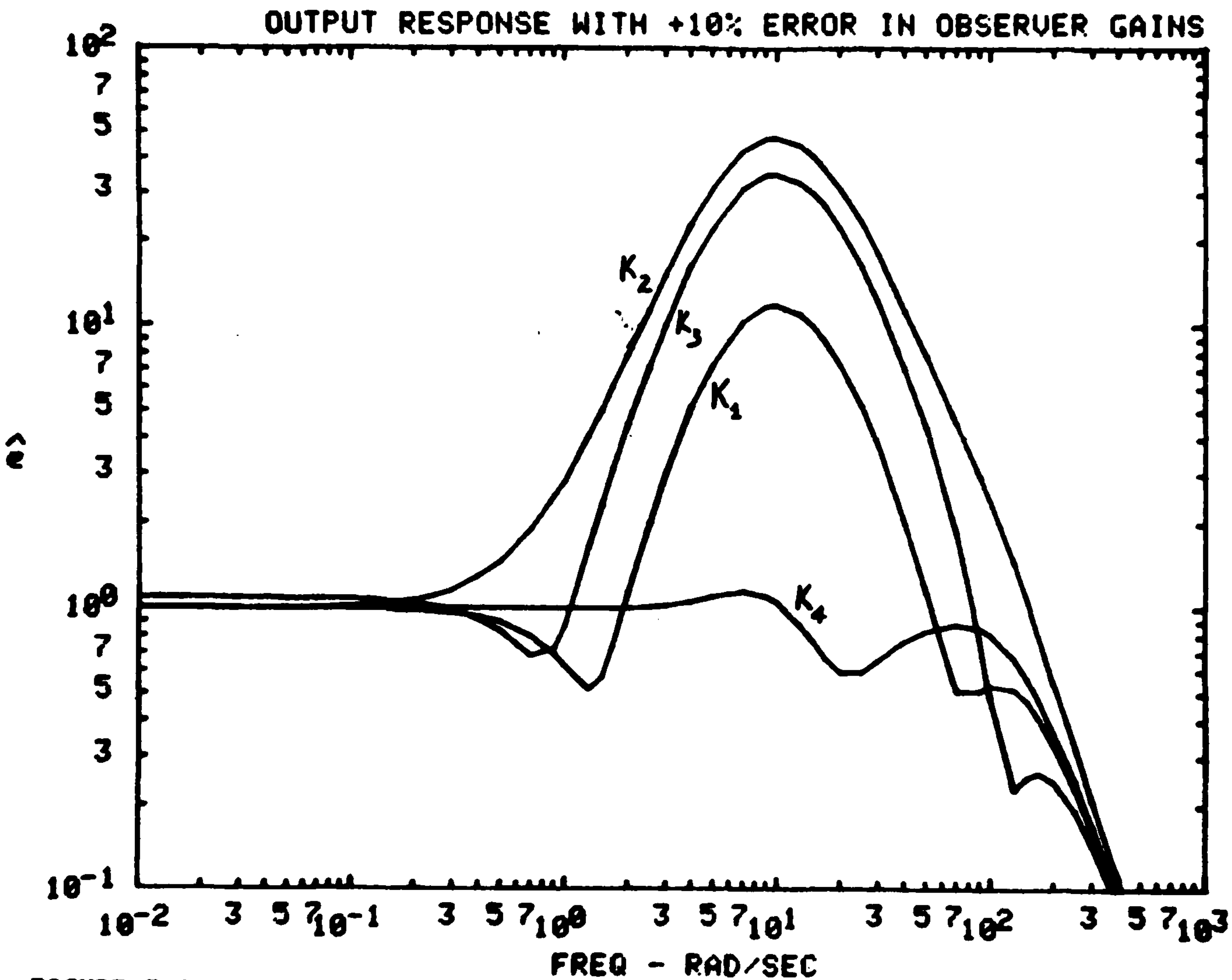

OUTPUT RESPONSE WITH +10% ERROR IN OBSERVER GAINS
$\hat{e}$
K_1
K_2
K_3
K_4
FREQ - RAD/SEC

FIGURE 7.1.5

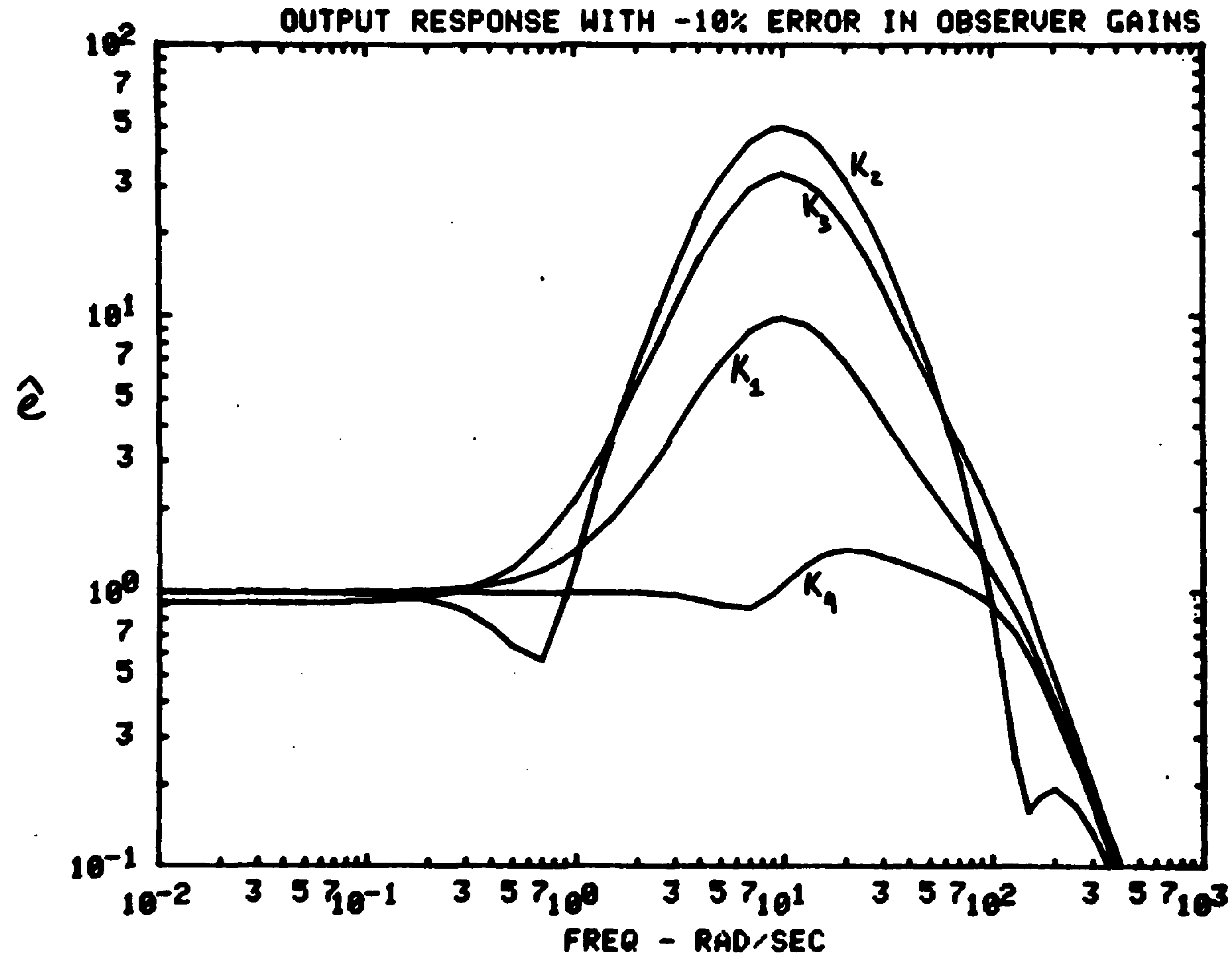
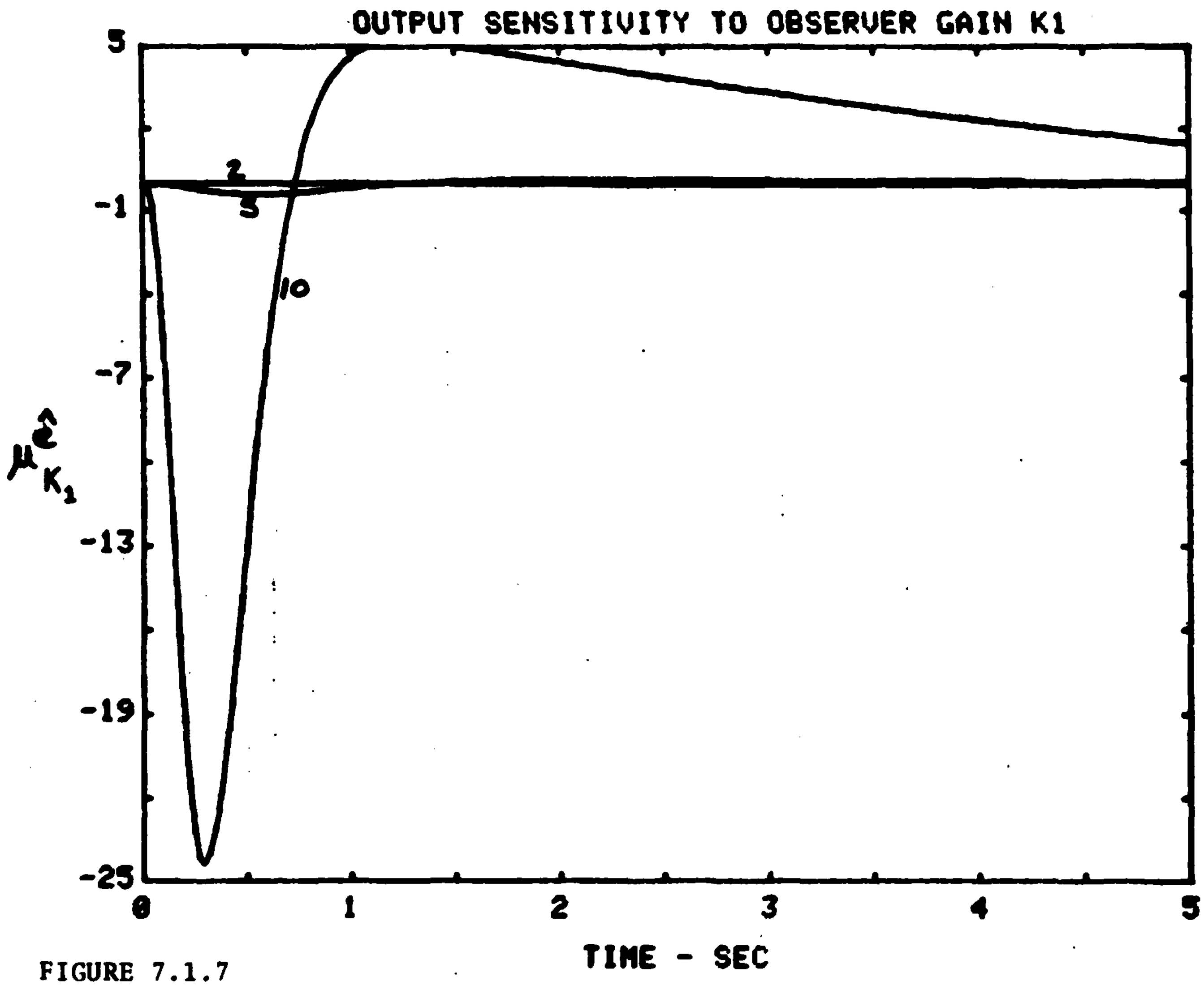

OUTPUT RESPONSE WITH -10% ERROR IN OBSERVER GAINS
ê
10^{2}
7
5
3
10^{1}
7
5
3
10^{0}
7
5
3
10^{-1}
K_2
K_3
K_1
K_4
10^{-2} 3 5 7 10^{-1} 3 5 7 10^{0} 3 5 7 10^{1} 3 5 7 10^{2} 3 5 7 10^{3}
FREQ - RAD/SEC

FIGURE 7.1.6



OUTPUT SENSITIVITY TO OBSERVER GAIN K1
$\mu^{\hat{e}}_{K_1}$
5
-1
-7
-13
-19
-25
0
1
2
3
4
5
TIME - SEC
2
5
10

FIGURE 7.1.7

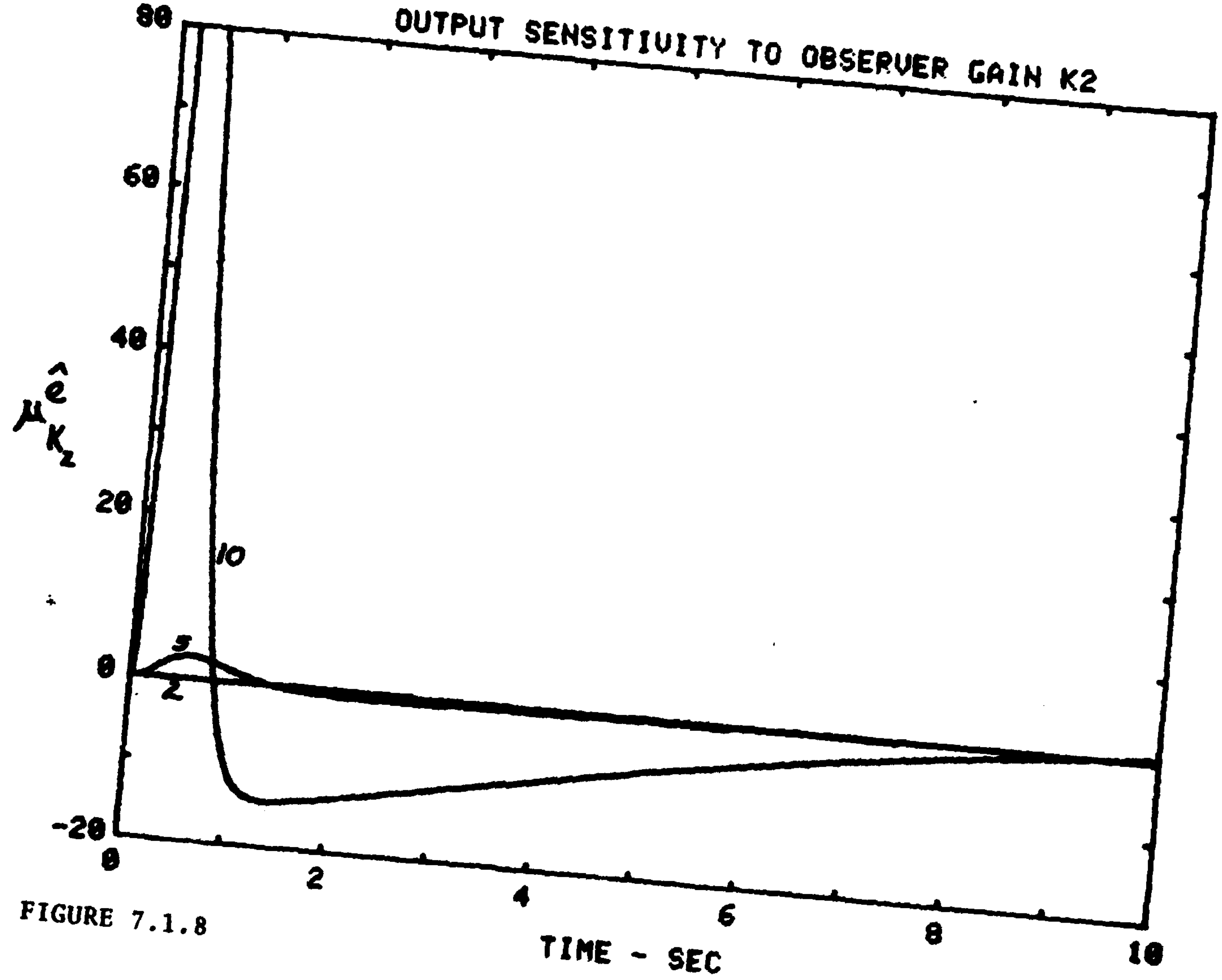

OUTPUT SENSITIVITY TO OBSERVER GAIN K2
80
60
40
20
0
-20
0
2
4
6
8
10
TIME - SEC
10
5
2

FIGURE 7.1.8

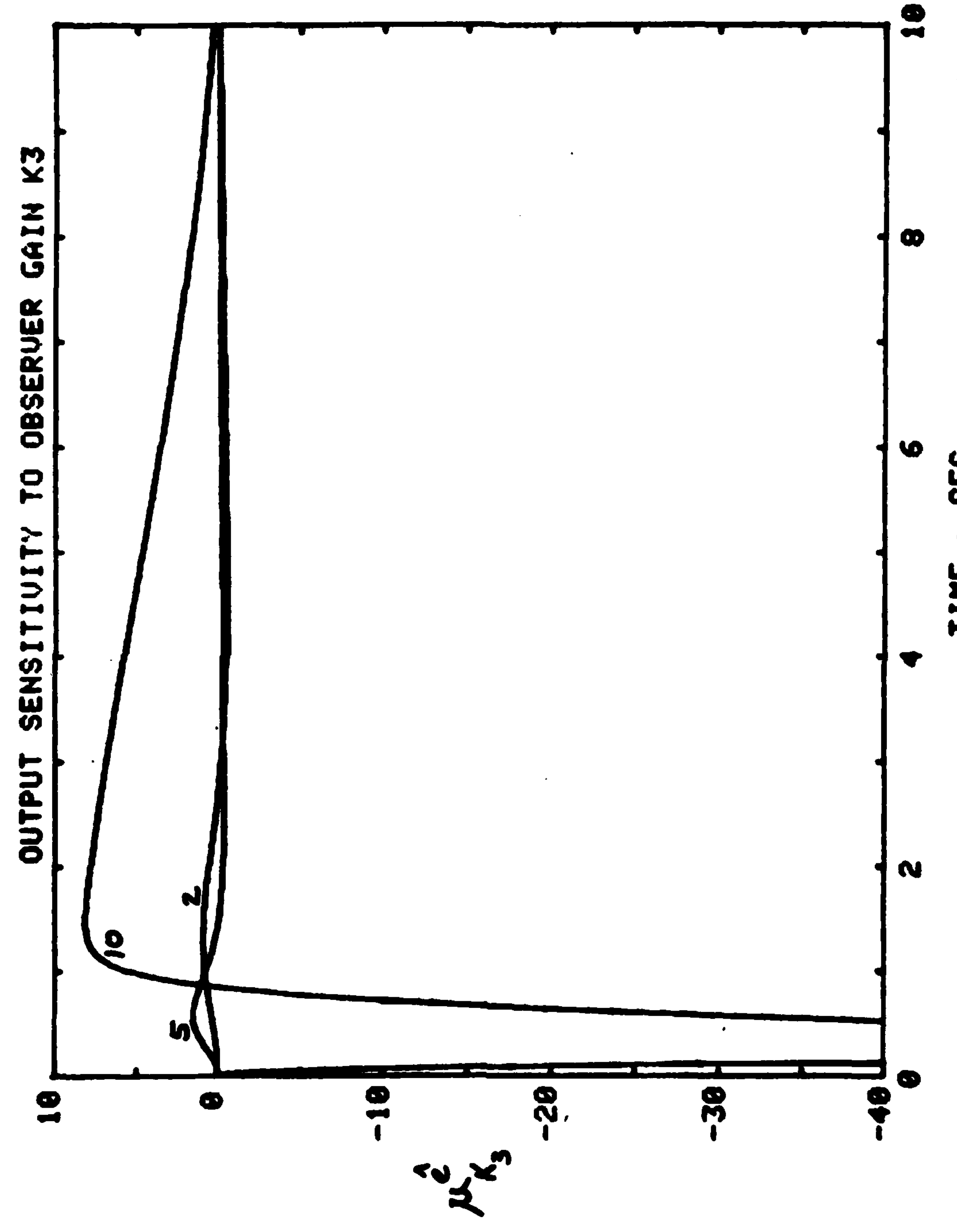

OUTPUT SENSITIVITY TO OBSERVER GAIN K3
10
0
-10
-20
-30
-40
0
2
4
6
8
10
TIME - SEC
10
5
2
$\mu^{\hat{e}}_{K_3}$

FIGURE 7.1.9

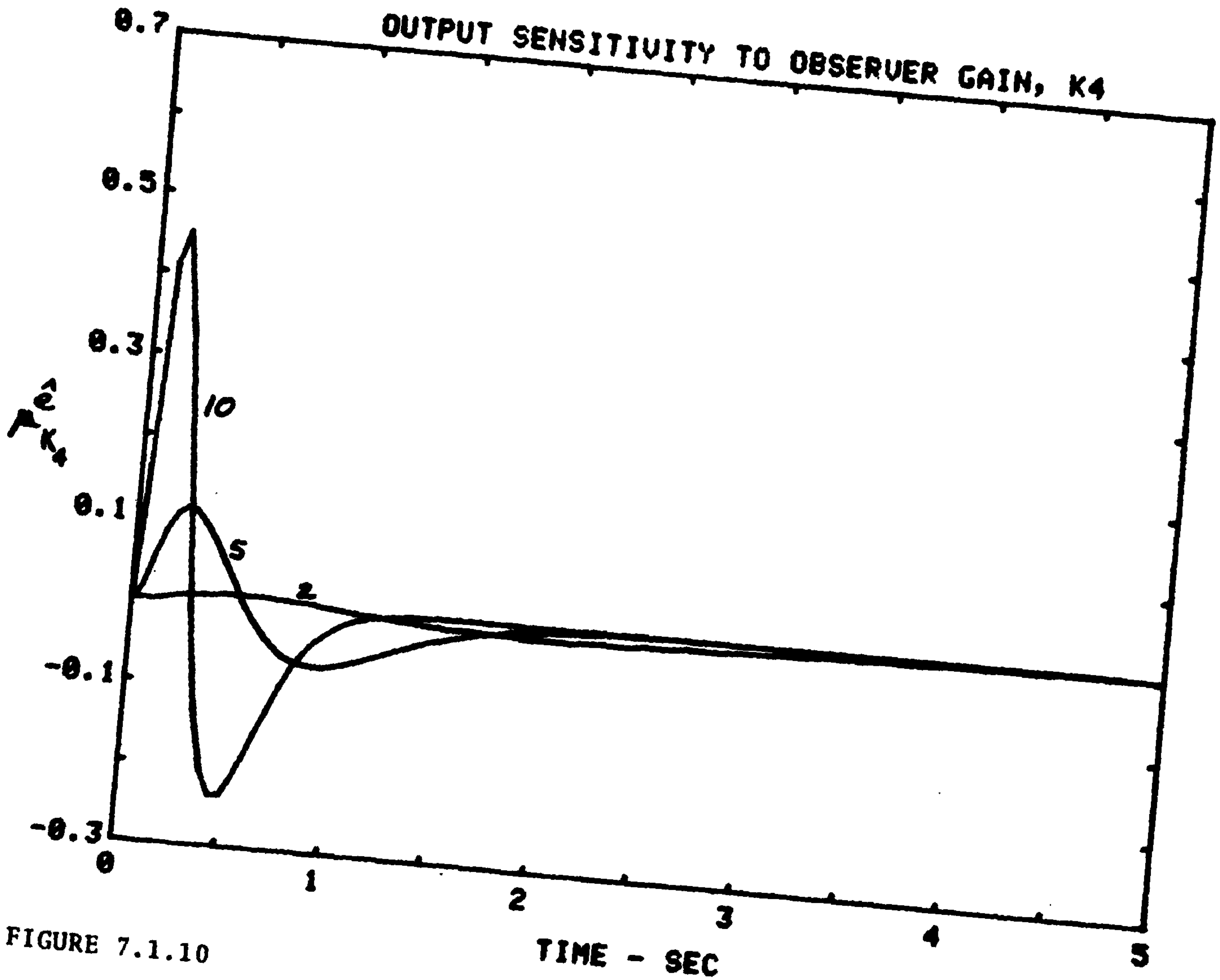

OUTPUT SENSITIVITY TO OBSERVER GAIN, K4
0.7
0.5
0.3
0.1
-0.1
-0.3
$\hat{e}_{K_4}$
10
5
2
0
1
2
3
4
5
TIME - SEC

FIGURE 7.1.10

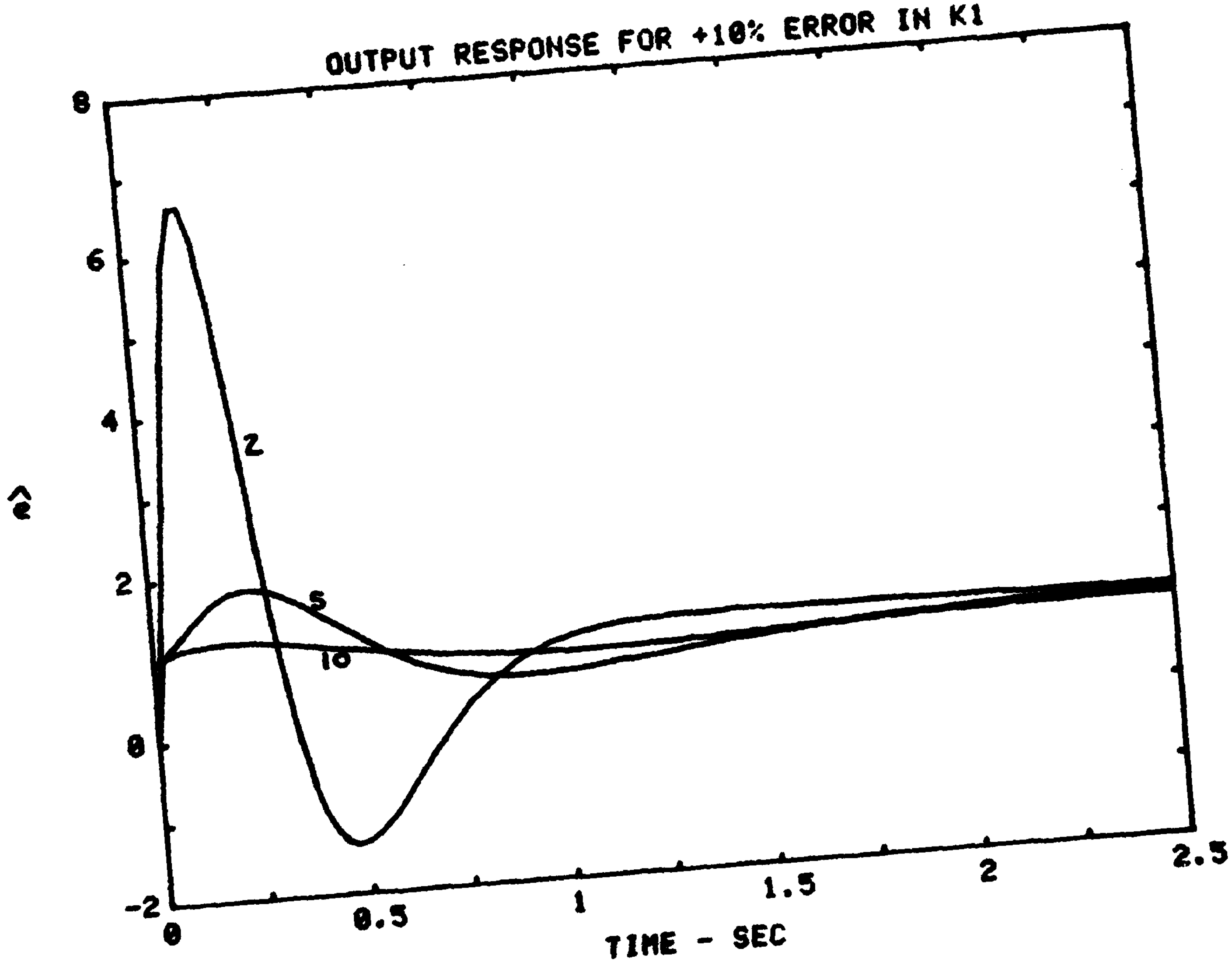

OUTPUT RESPONSE FOR +10% ERROR IN K1
8
6
4
2
0
-2
0
0.5
1
1.5
2
2.5
TIME - SEC
2
5
10

FIGURE 7.1.11

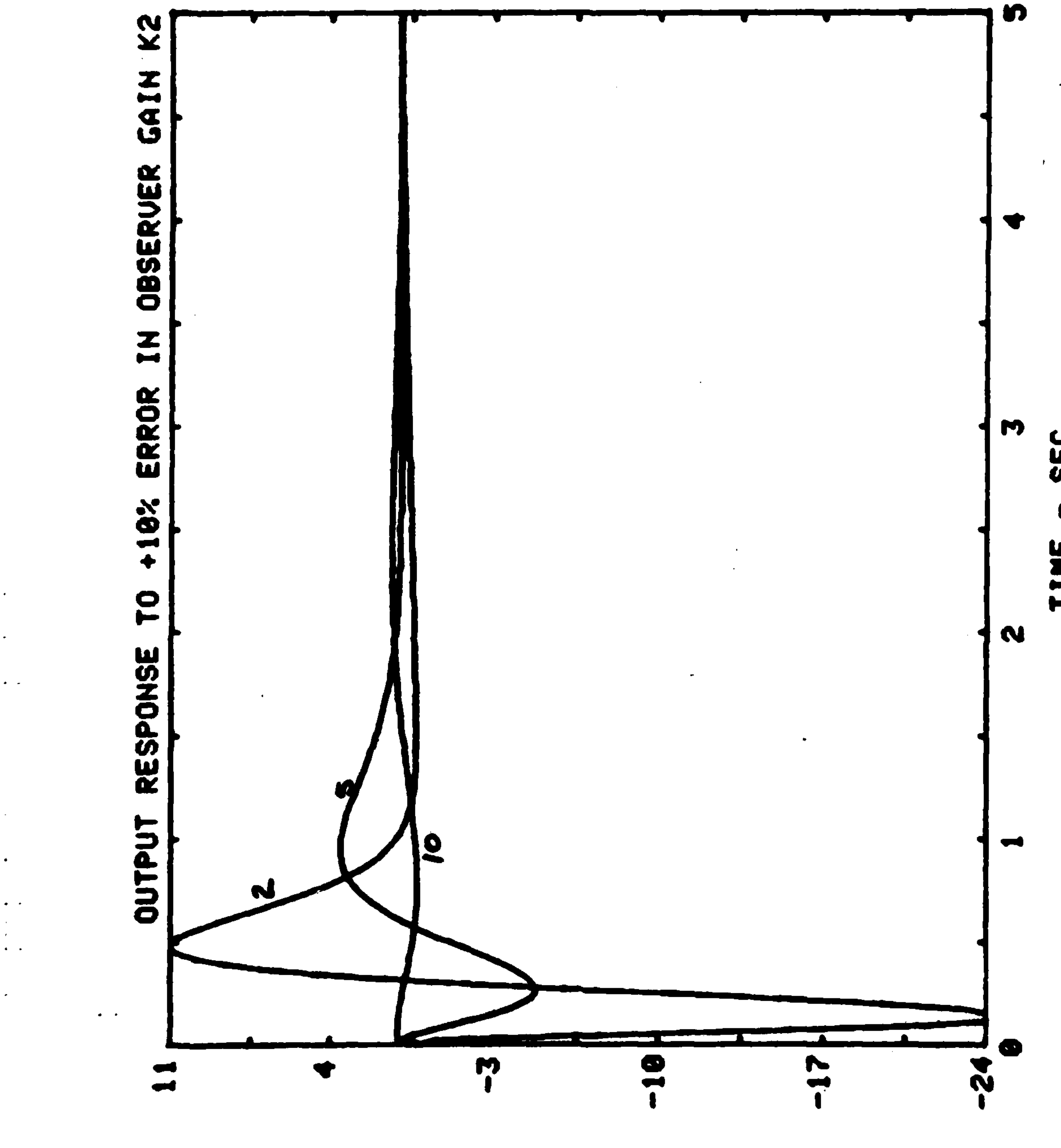

OUTPUT RESPONSE TO +10% ERROR IN OBSERVER GAIN K2
11
4
-3
-10
-17
-24
0
1
2
3
4
5
TIME - SEC
2
5
10

FIGURE 7.1.12

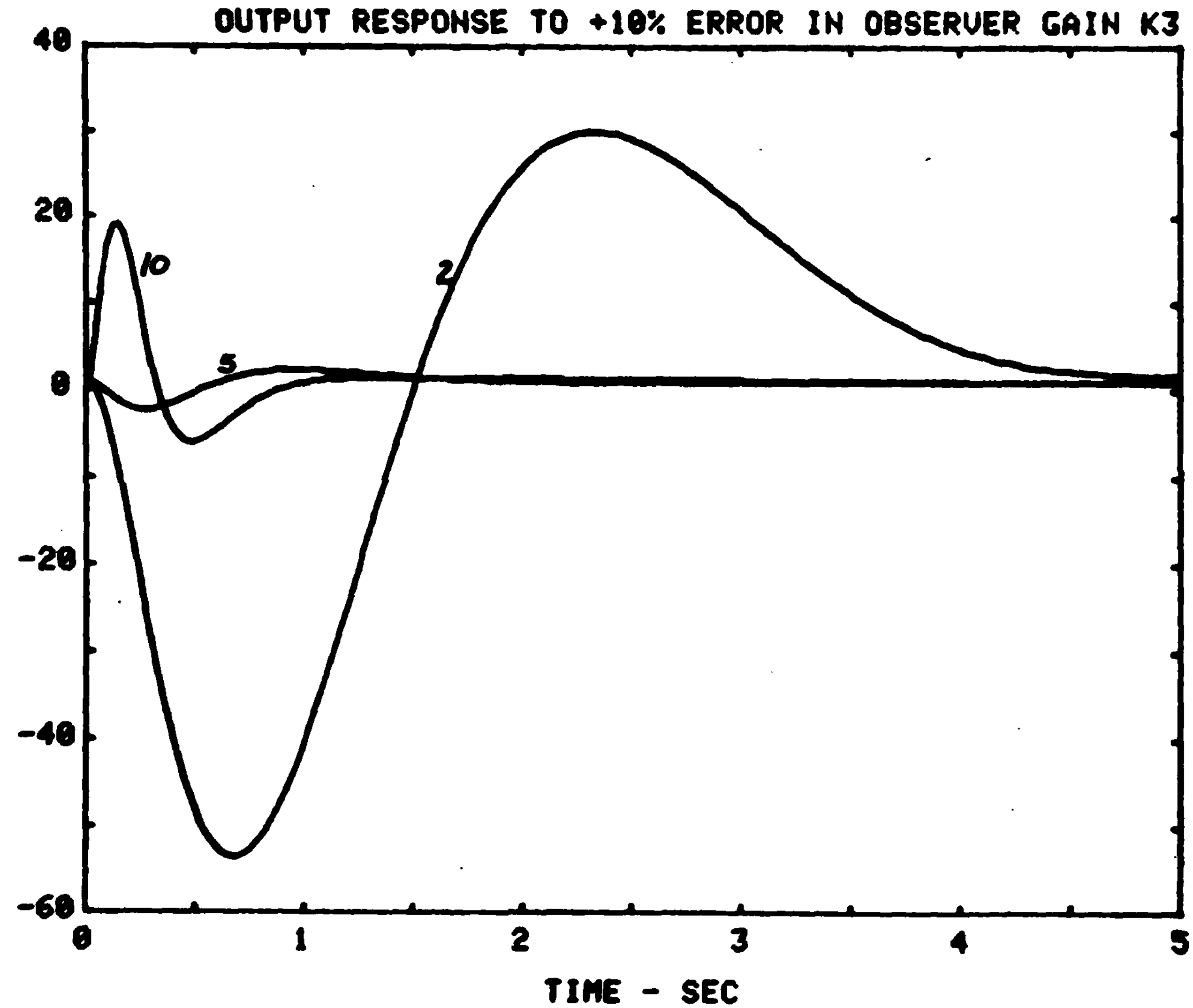
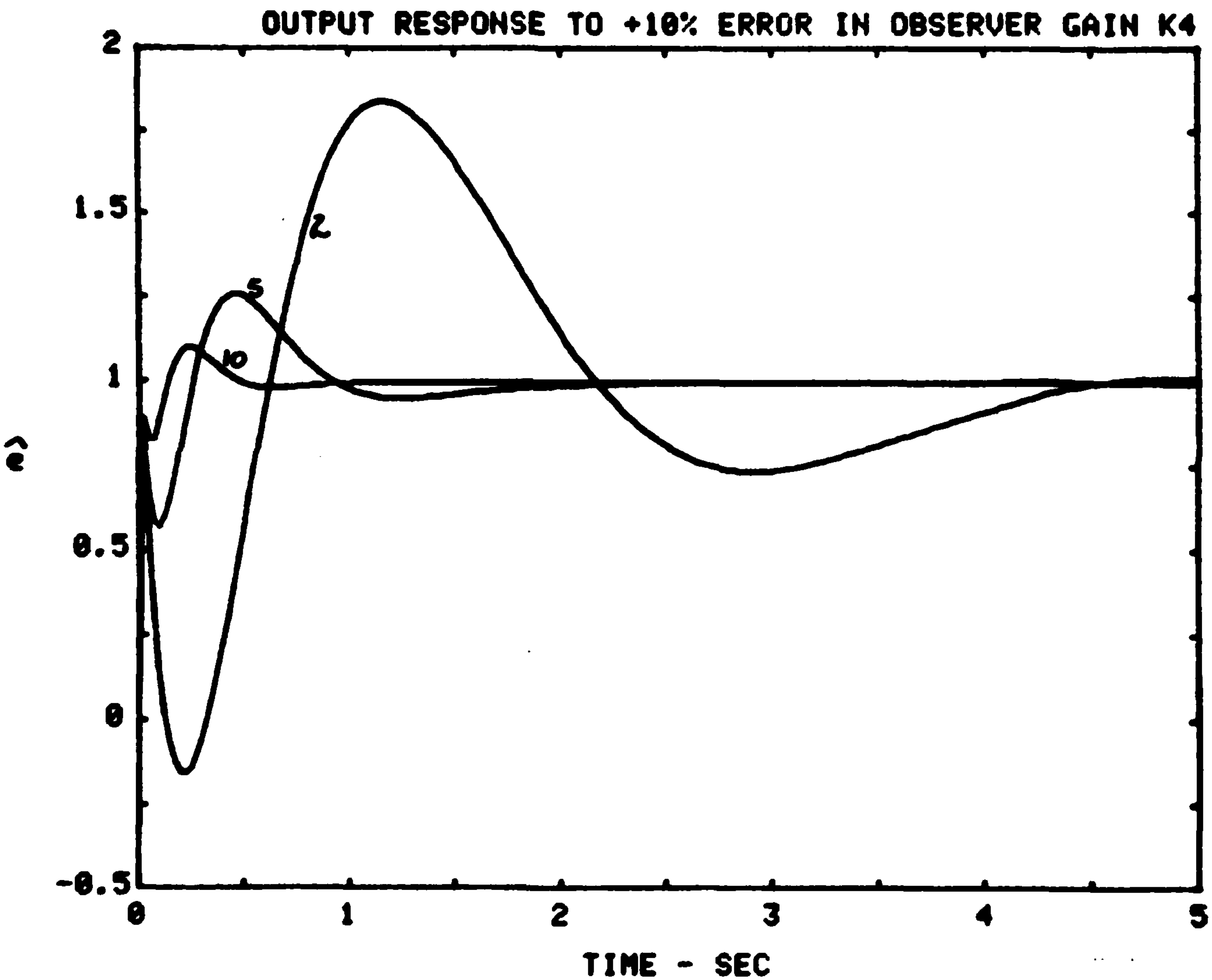

OUTPUT RESPONSE TO +10% ERROR IN OBSERVER GAIN K3
40
20
0
-20
-40
-60
ê
10
5
2
0
1
2
3
4
5
TIME - SEC

FIGURE 7.1.13



OUTPUT RESPONSE TO +10% ERROR IN OBSERVER GAIN K4
2
1.5
1
0.5
0
-0.5
0
1
2
3
4
5
TIME - SEC
ê
2
5
10

FIGURE 7.1.14

8.0 OUTPUT SENSITIVITY WITH RESPECT TO THE INPUTS

8.1 Relations of the Inputs

Referring to Figure 2.2.1(a) the observer has as an input the signal θ because $\dot{\theta}$ is unmeasurable; in fact, the function of the sensor is to generate a signal equal to $\dot{\theta}$.

In order to study the effects of the other parameters on the output response, it has been assumed that $\theta = \int \dot{\theta} dt$. In actual fact, the signal θ is instrument generated and so may deviate from the ideal mathematical relation.

8.2 Sensitivity with Respect to θ

From Figure 2.2.1(a) it is seen that θ only enters into the observer equation as

$$\hat{e} = M(sI-F)^{-1}(B\theta + \underline{K}e_o) \qquad (8.2.1)$$

and so

$$\frac{\partial \hat{e}}{\partial \theta} = M(sI-F)^{-1}B$$

from which

$$\mu_{\theta}^{\hat{e}} = M(sI-F)^{-1}B\theta \qquad (8.2.2)$$

M and B are known from equations (2.1.7) and (2.7.5) and hence rather simply one finds

$$\mu_{\theta}^{\hat{e}} = \frac{a}{\Delta_F} \left(m_2 f_{24} + m_3 f_{34} + m_4 f_{44}\right)\theta \qquad (8.2.3)$$

which reduces to

$$\mu_{\theta}^{\hat{e}} = \frac{a}{\Delta_F} \left[m_4\Delta_{A1} - (m_3K_3 + m_2K_2)s - m_2K_3\right]\dot{\theta} \qquad (8.2.4)$$

Comparing these equations to (6.3.3) and 6.3.4), it is seen that the sensitivity due to variations in θ are identical to variations in the observer forward path gain. If one thinks in terms of a transfer function from θ to the observer output and that a is the transfer function gain, then this result is obvious.

8.3 Results

The results of section 6.3 apply directly to this case. In particular note Figures 6.3.1 and 6.3.4. These represent the response of $\hat{\theta}$ with a 10% error in either observer gain a or θ. This is a constant, fixed gain or offset. The observer output, $\hat{\theta}$ is off, and there are some additional transients, but the basic tracking ability still exists.

A problem that could arise, and must be considered is the effect of noise on $\underset{\sim}{\theta}$. Figure 6.3.1 indicates that the output is very sensitive to this noise in the frequency ranges of 0.1 to 100 rad/sec.

Figure 8.3.1 is a solution of the state equations for $\hat{\theta}$. The observer roots are all at -10 and the input θ is taken to be

$$\theta = \int \dot{\theta} dt + 0.01\sin(10t) \qquad (8.4.1)$$

This means that θ is the true integral of $\dot{\theta}$ but has some 10 rad/sec noise superimposed. The effect of this small magnitude noise is apparent.

Figure 8.3.2 illustrates $\hat{\theta}$ under the same conditions except that θ has a random signal of magnitude .01 superimposed.

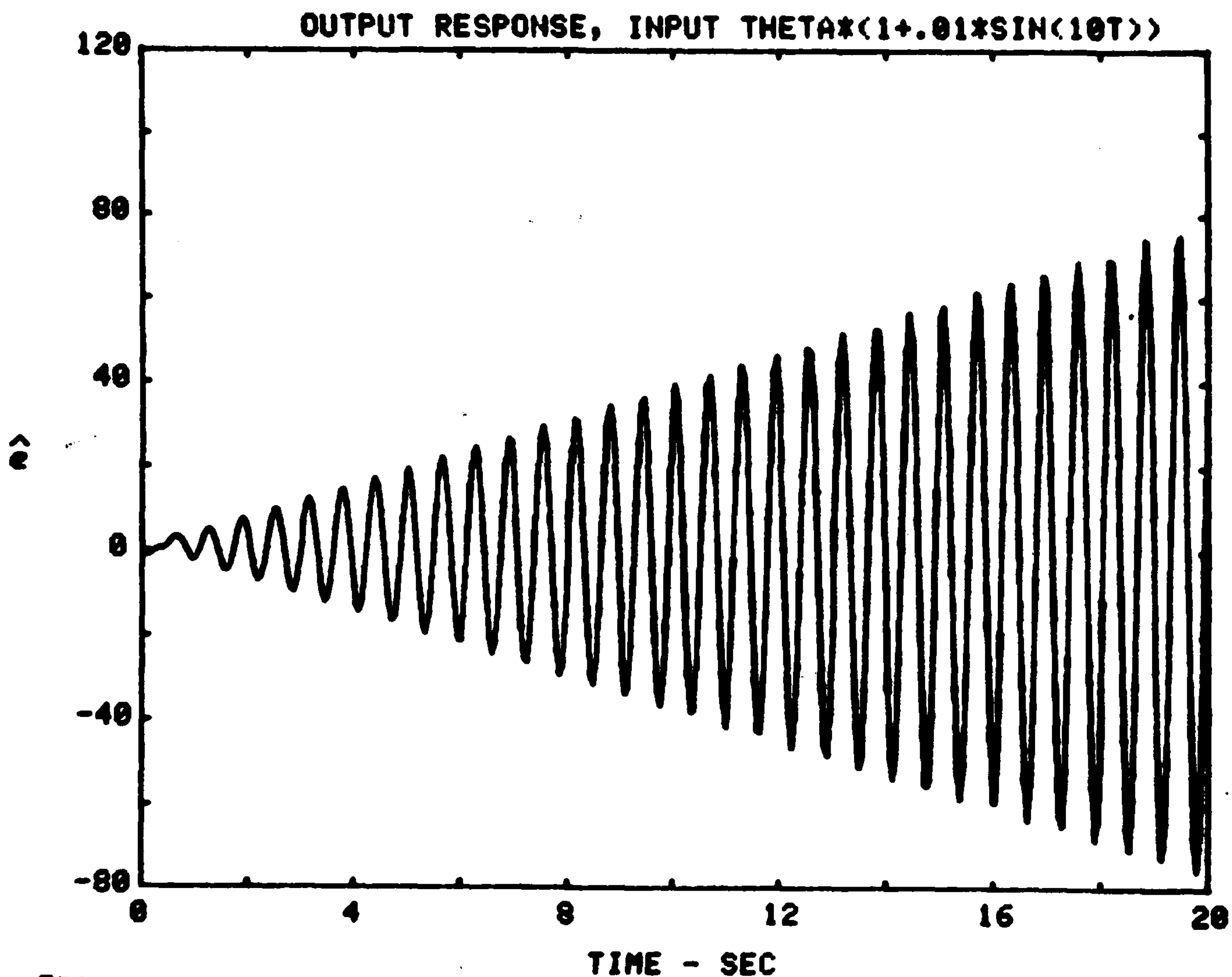

OUTPUT RESPONSE, INPUT THETA*(1+.01*SIN(10T))
120
80
40
0
-40
-80
θ̂
0
4
8
12
16
20
TIME - SEC

FIGURE 8.3.1

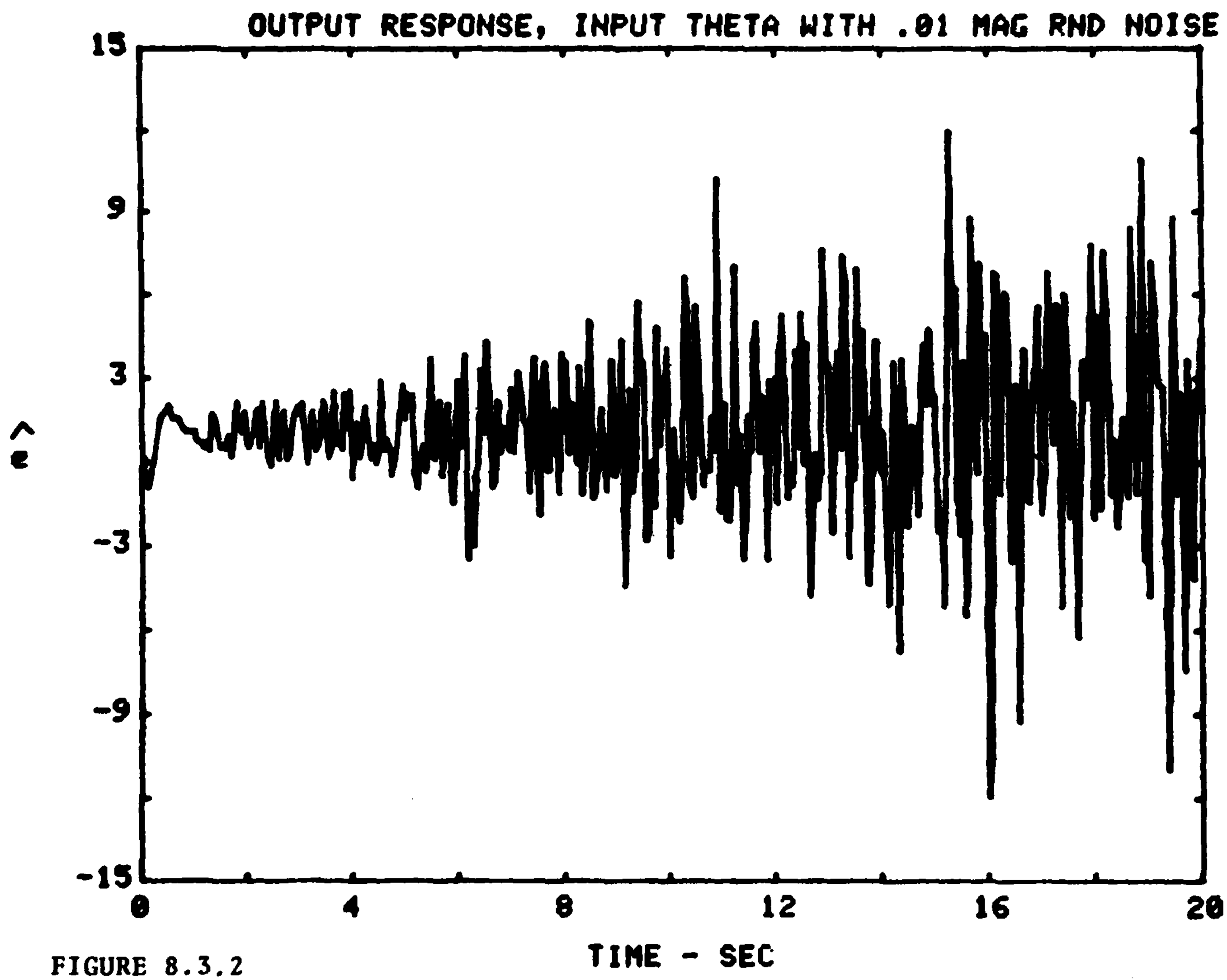

OUTPUT RESPONSE, INPUT THETA WITH .01 MAG RND NOISE
15
9
3
-3
-9
-15
$\hat{e}$
0
4
8
12
16
20
TIME - SEC

FIGURE 8.3.2

9.0 OUTPUT SENSITIVITY WITH RESPECT TO PARAMETERS OF THE SENSOR

9.1 Sensitivity Equations

From Figure 2.2.1(b) the rate sensor equation is

$$e_o = C_4(sI-A)^{-1}B\theta \tag{9.1.1}$$

and the observer output is given as

$$\hat{e} = M(sI-F)^{-1}(B\theta + \underline{K}e_o) \tag{9.1.2}$$

The parameters of concern are those in the A matrix of (9.1.1). There are four parameters, δ_1, ω_1, δ_2 and ω_2. Denote these as the general parameter p, then by the chain rule

$$\frac{\partial \hat{e}}{\partial p} = \frac{\partial \hat{e}}{\partial e_o} \frac{\partial e_o}{\partial A} \frac{\partial A}{\partial p} \tag{9.1.3}$$

Application of (9.1.3) gives

$$\frac{\partial \hat{e}}{\partial p} = M(sI-F)^{-1} \underline{K} \frac{\partial e_o}{\partial p}$$

$$\frac{\partial \hat{e}}{\partial p} = M(sI-F)^{-1} \underline{K} C_4 (sI-A)^{-1} \frac{\partial A}{\partial p} (sI-A)^{-1}B\theta \tag{9.1.4}$$

The A matrix is

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -b & -c & -d & -e \end{bmatrix}$$

Where b, c, d, and e are functions of the parameters.

Therefore,

$$\frac{\partial A}{\partial p} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{\partial b}{\partial p} & -\frac{\partial c}{\partial p} & -\frac{\partial d}{\partial p} & -\frac{\partial e}{\partial p} \end{bmatrix}$$

The B matrix of (9.1.4) is

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ a \end{bmatrix}$$

and the $(sI-A)^{-1}$ matrix is given in equation (2.3.6).

The product is

$$(sI-A)^{-1}B = \begin{bmatrix} 1 \\ s \\ s^2 \\ s^3 \end{bmatrix} \frac{a}{\Delta_A} \tag{9.1.6}$$

and hence the product of

$$\frac{\partial A}{\partial p} (sI-A)^{-1}B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ r \end{bmatrix} \frac{a}{\Delta_A} \tag{9.1.7}$$

and $$r = -\left(\frac{\partial b}{\partial p} + s\frac{\partial c}{\partial p} + s^2 \frac{\partial d}{\partial p} + s^3 \frac{\partial e}{\partial p}\right) \tag{9.1.8}$$

Since the $\frac{\partial}{\partial p}$ may be removed, (9.1.8) becomes

$$r = - \frac{\partial}{\partial p} (b + sc + s^2 d + s^3 e) \tag{9.1.9}$$

The characteristic equation of the sensor is

$$\Delta_A = s^4 + s^3 e + s^2 d + sc + b$$

and so (9.1.9) may be written as

$$r = - \frac{\partial}{\partial p} \Delta_A$$

and (9.1.7) will then become

$$\frac{\partial A}{\partial p} (sI-A)^{-1}B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\frac{\partial}{\partial p} \Delta_A \end{bmatrix} \frac{a}{\Delta_A} \tag{9.1.10}$$

Next,

$$C_4(sI-A)^{-1} = \frac{1}{\Delta_A} [a_{41} \; a_{42} \; a_{43} \; s^3]$$

so that

$$C_4(sI-A)^{-1} \frac{\partial A}{\partial p} (sI-A)^{-1}B = \frac{as^3}{\Delta_A^2} \frac{\partial}{\partial p} (-\Delta_A) \qquad (9.1.11)$$

The $(sI-F)^{-1}$ term has the form

$$(sI-F)^{-1} = \begin{bmatrix} f_{11} & f_{12} & f_{13} & f_{14} \\ f_{21} & f_{22} & f_{23} & f_{24} \\ f_{31} & f_{32} & f_{33} & f_{34} \\ f_{41} & f_{42} & f_{43} & f_{44} \end{bmatrix}$$

and since $m = [o \; m_2 \; m_3 \; m_4]$ then

$$m(sI-F)^{-1}\underline{K} = \frac{1}{\Delta_f} [\Sigma m_i f_{i1} \quad \Sigma m_i f_{i2} \quad \Sigma m_i f_{i3} \quad \Sigma m_i f_{i4}] \begin{bmatrix} K_1 \\ K_2 \\ K_3 \\ K_4 \end{bmatrix}_{i=2-4} \qquad (9.1.12)$$

or

$$(sI-F)^{-1}\underline{K} = \frac{1}{\Delta_F} \sum_{2=j}^{4} (K_j \sum_{i=2}^{4} m_i f_{ij}) \qquad (9.1.13)$$

Combining equations (9.1.11), (9.1.13) and the fact that $\mu_p^{\hat{e}} = p\partial\hat{e}/\partial p$ produces the desired result

$$\mu_p^{\hat{e}} = (p\frac{\partial}{\partial p}\Delta_A)(\frac{-as^2}{\Delta_A^2\Delta_F}) \; [\sum_{j=2}^{4} (K_j \sum_{i=2}^{4} m_i f_{ij})]\dot{\theta} \qquad (9.1.14)$$

The term $p \frac{\partial}{\partial p} \Delta_A$ depends upon which of the parameters is considered.

The characteristic equation of the rate sensor was given in equation (2.1.13) as

$$\Delta_A = \Delta_{A1}\Delta_{A2} = (s^2+2\delta_1\omega_1 s+\omega_1^2)\ (s^2+2\delta_2\omega_2 s+\omega_2^2)$$

Therefore, for δ_1;

$$p\frac{\partial}{\partial p}\Delta_A = \delta_1[2\omega_1 s]\ \Delta_{A2} \tag{9.1.15}$$

for ω_1;

$$p\frac{\partial}{\partial p}\Delta_A = \omega_1[2\delta_1 s+2\omega_1]\ \Delta_{A2} \tag{9.1.16}$$

for δ_2;

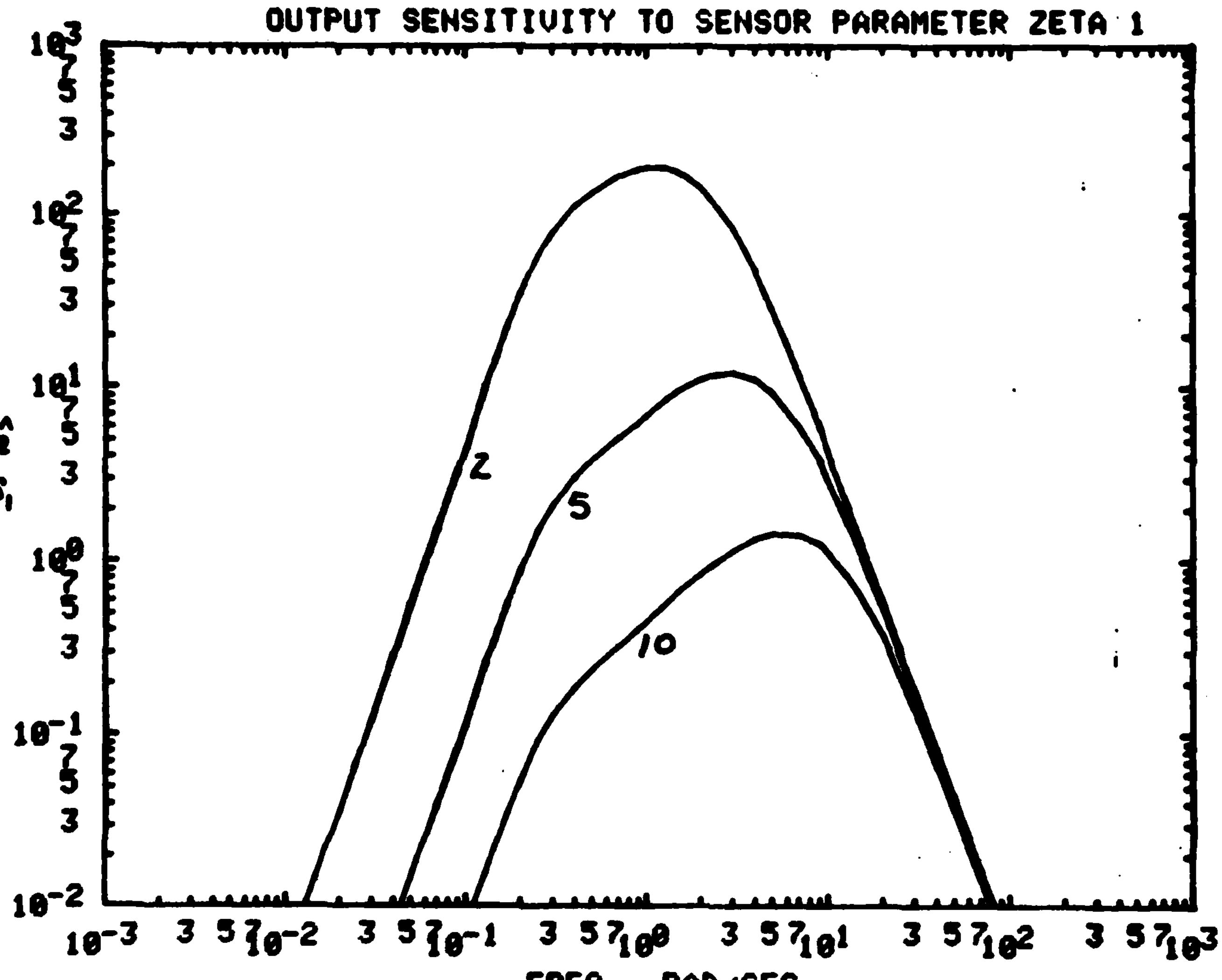
$$p\frac{\delta}{\delta p}\Delta_A = \delta_2[2\omega_2 s]\ \Delta_{A1} \tag{9.1.17}$$

and for ω_2;

$$p\frac{\partial}{\partial p}\Delta_A = \omega_2[2\delta_2 s + 2\omega_2]\ \Delta_{A1} \tag{9.1.18}$$

Figures 9.1.1-4 illustrate the sensitivity of $\hat{e}$ with respect to δ_1, ω_1, δ_2 and ω_2 respectively. As expected, parameter sensitivity decreases as observer gains $\underline{K}$ increase. The output $\hat{e}$ is sensitivity to ω_2, where ω_2 is the high corner frequency of the rate sensor itself. As seen from equation (9.1.4) none of the parameters have any steady state effect even when $\dot{\theta}$ is a ramp.

Figures 9.1.5-8 illustrate the time solution of equation (9.1.14) for δ_1, ω_1, δ_2 and ω_2 respectively. As is



OUTPUT SENSITIVITY TO SENSOR PARAMETER ZETA 1
$\hat{\mu}^{e}_{\delta_1}$
FREQ - RAD/SEC
2
5
10

FIGURE 9.1.1

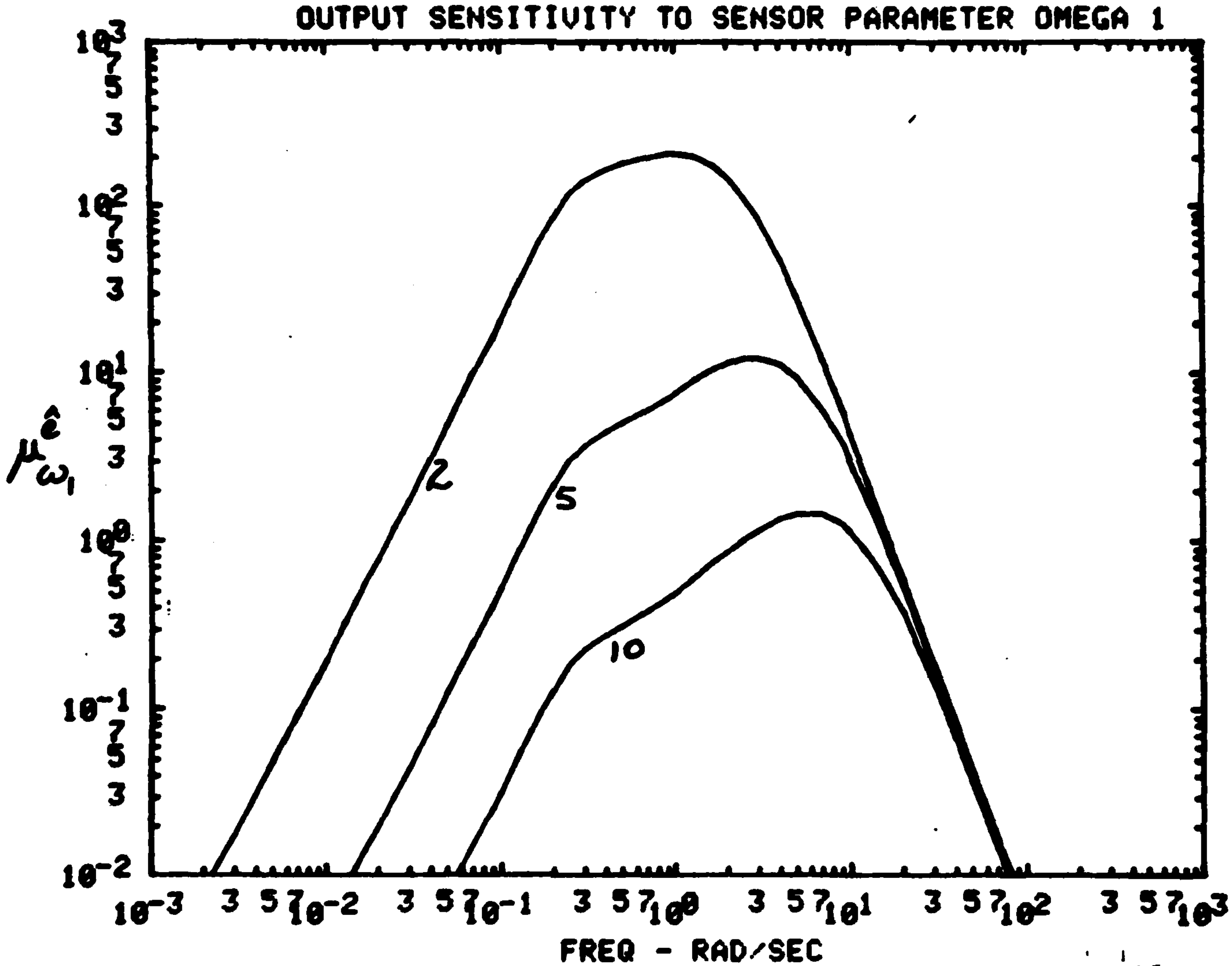

OUTPUT SENSITIVITY TO SENSOR PARAMETER OMEGA 1
$\mu^{\hat{e}}_{\omega_1}$
FREQ - RAD/SEC
2
5
10

FIGURE 9.1.2

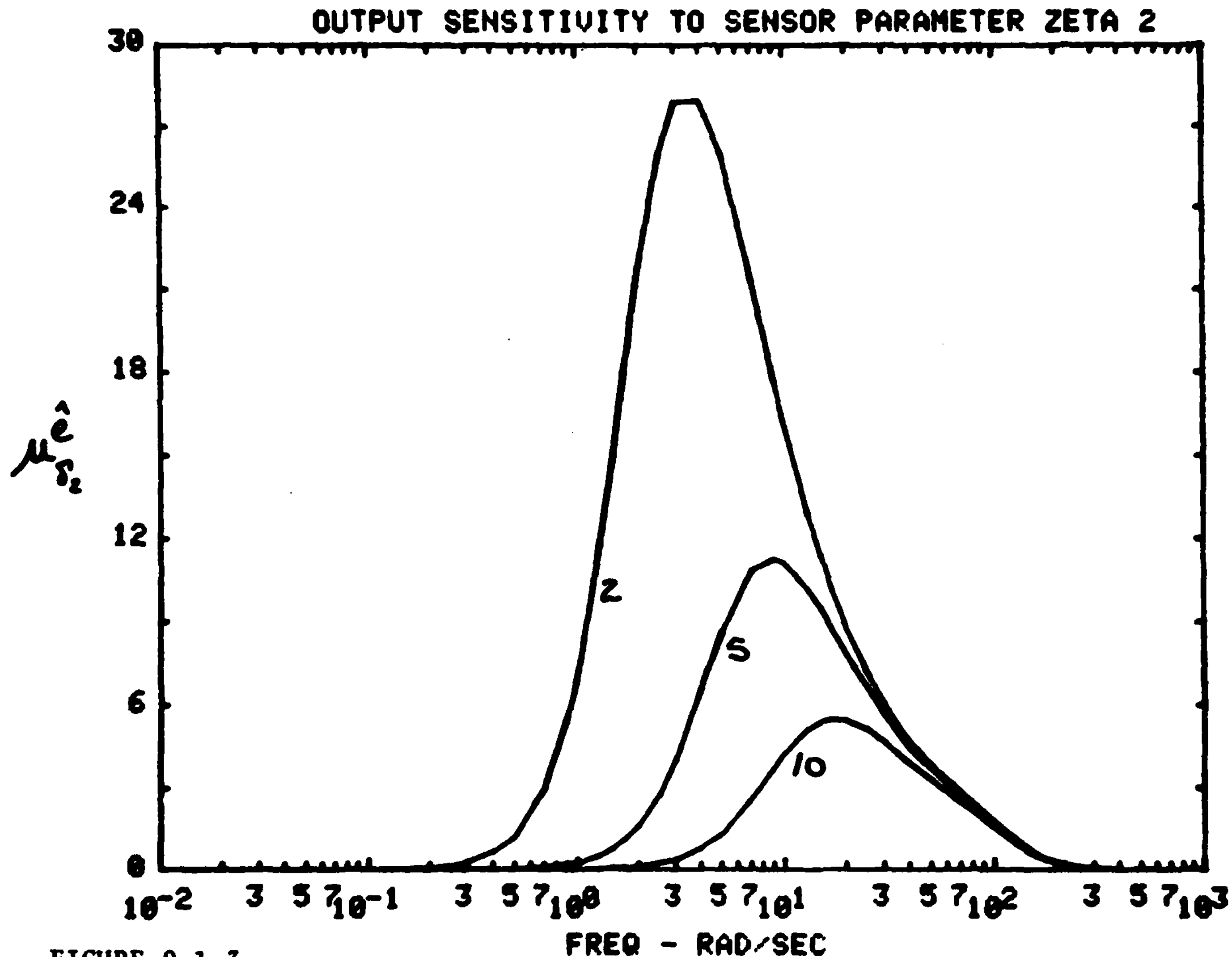

OUTPUT SENSITIVITY TO SENSOR PARAMETER ZETA 2
30
24
18
12
6
0
$\mu^{\hat{e}}_{\zeta_2}$
2
5
10
10^{-2} 3 5 7 10^{-1} 3 5 7 10^{0} 3 5 7 10^{1} 3 5 7 10^{2} 3 5 7 10^{3}
FREQ - RAD/SEC

FIGURE 9.1.3

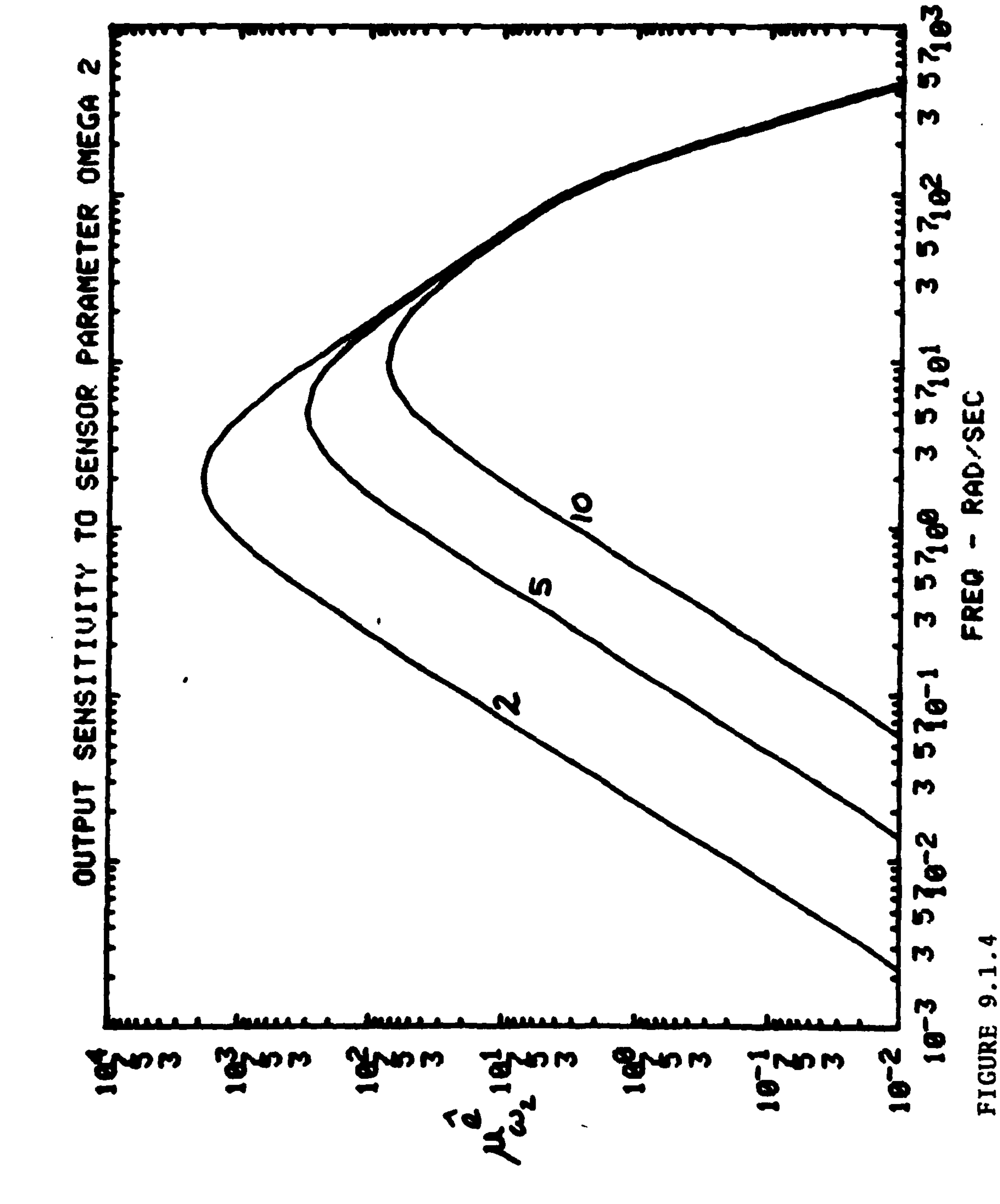
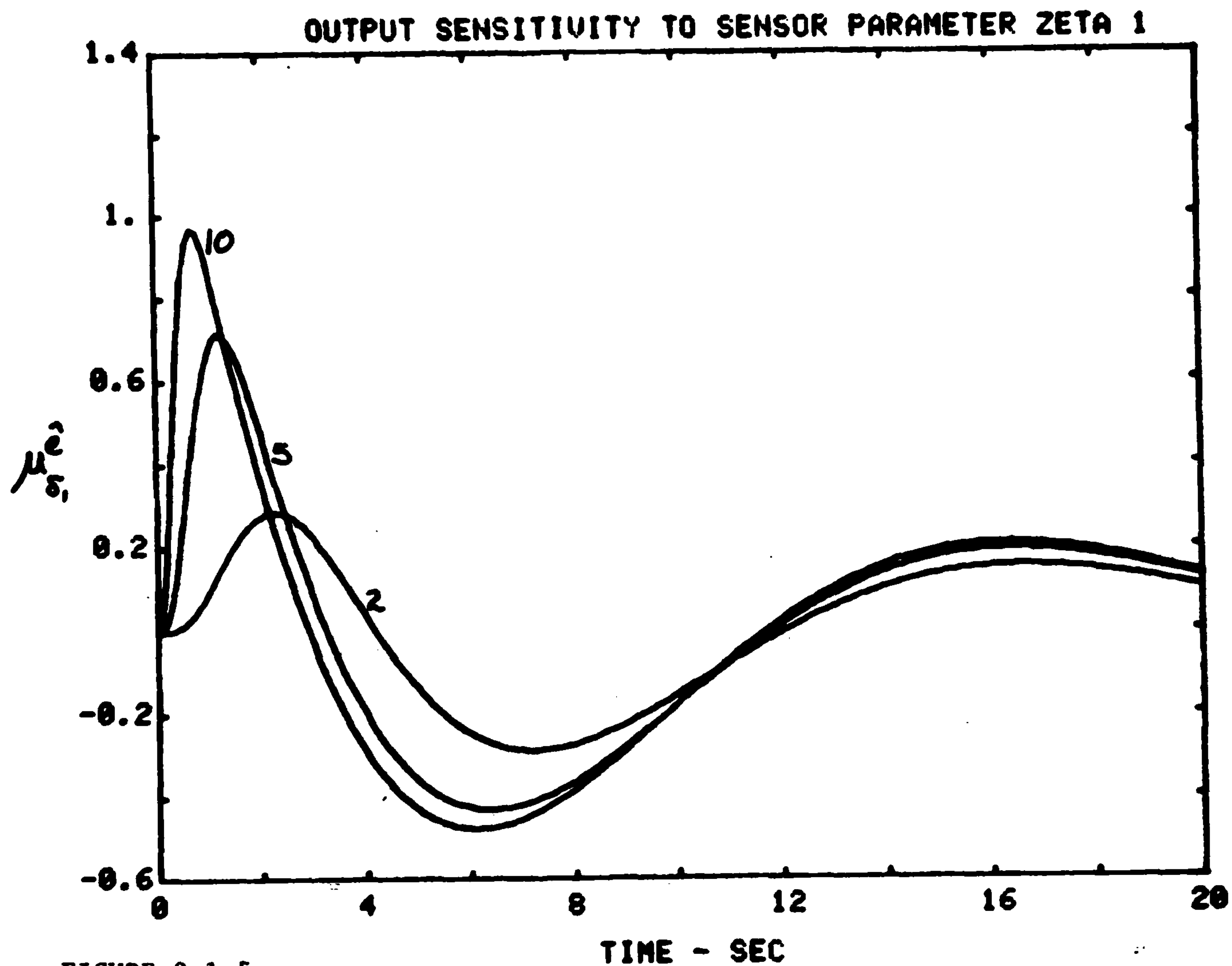

OUTPUT SENSITIVITY TO SENSOR PARAMETER OMEGA 2
FREQ - RAD/SEC
2
5
10

FIGURE 9.1.4



OUTPUT SENSITIVITY TO SENSOR PARAMETER ZETA 1
1.4
1.
0.6
0.2
-0.2
-0.6
0
4
8
12
16
20
TIME - SEC
10
5
2
$\mu^{\hat{e}}_{\delta_1}$

FIGURE 9.1.5

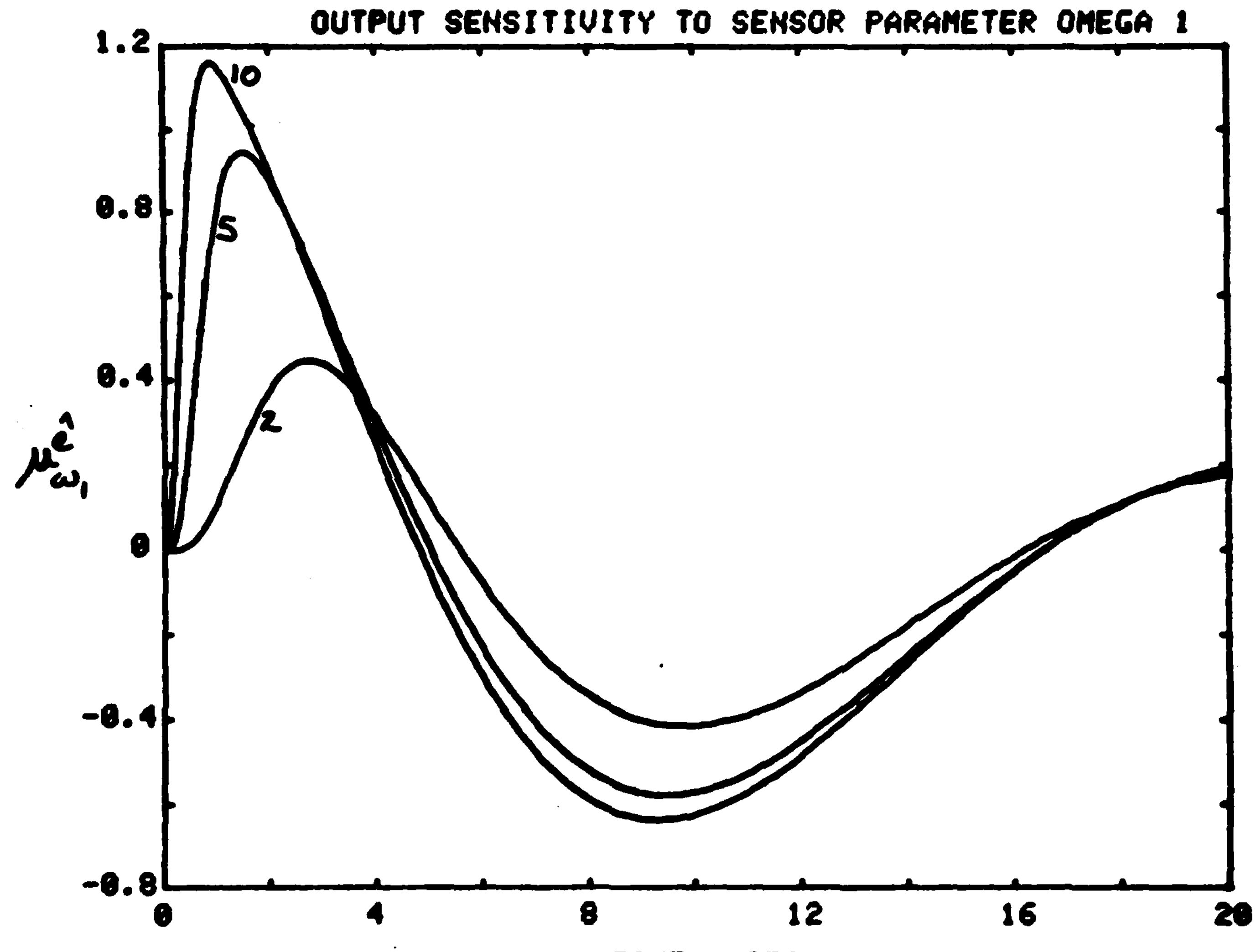
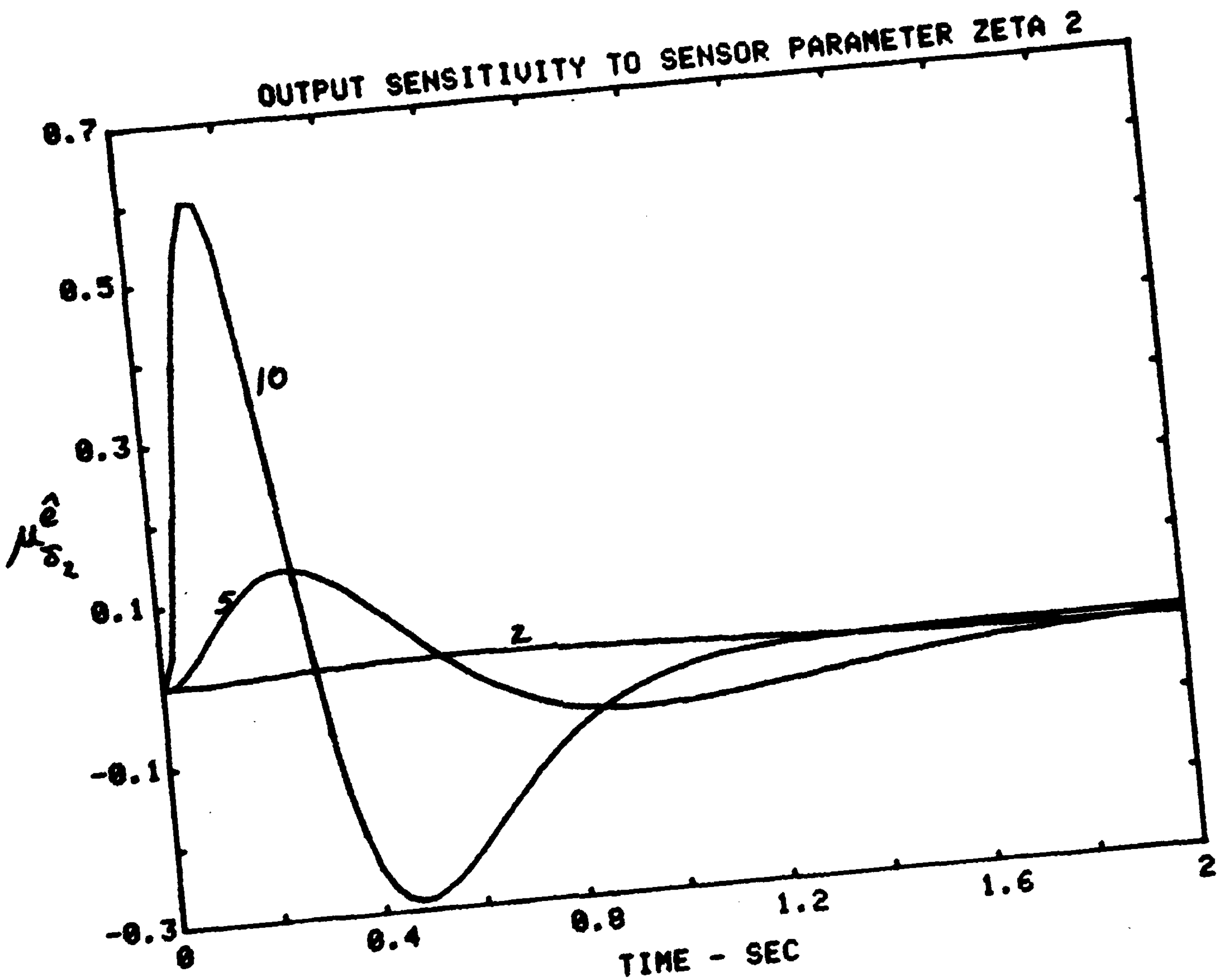
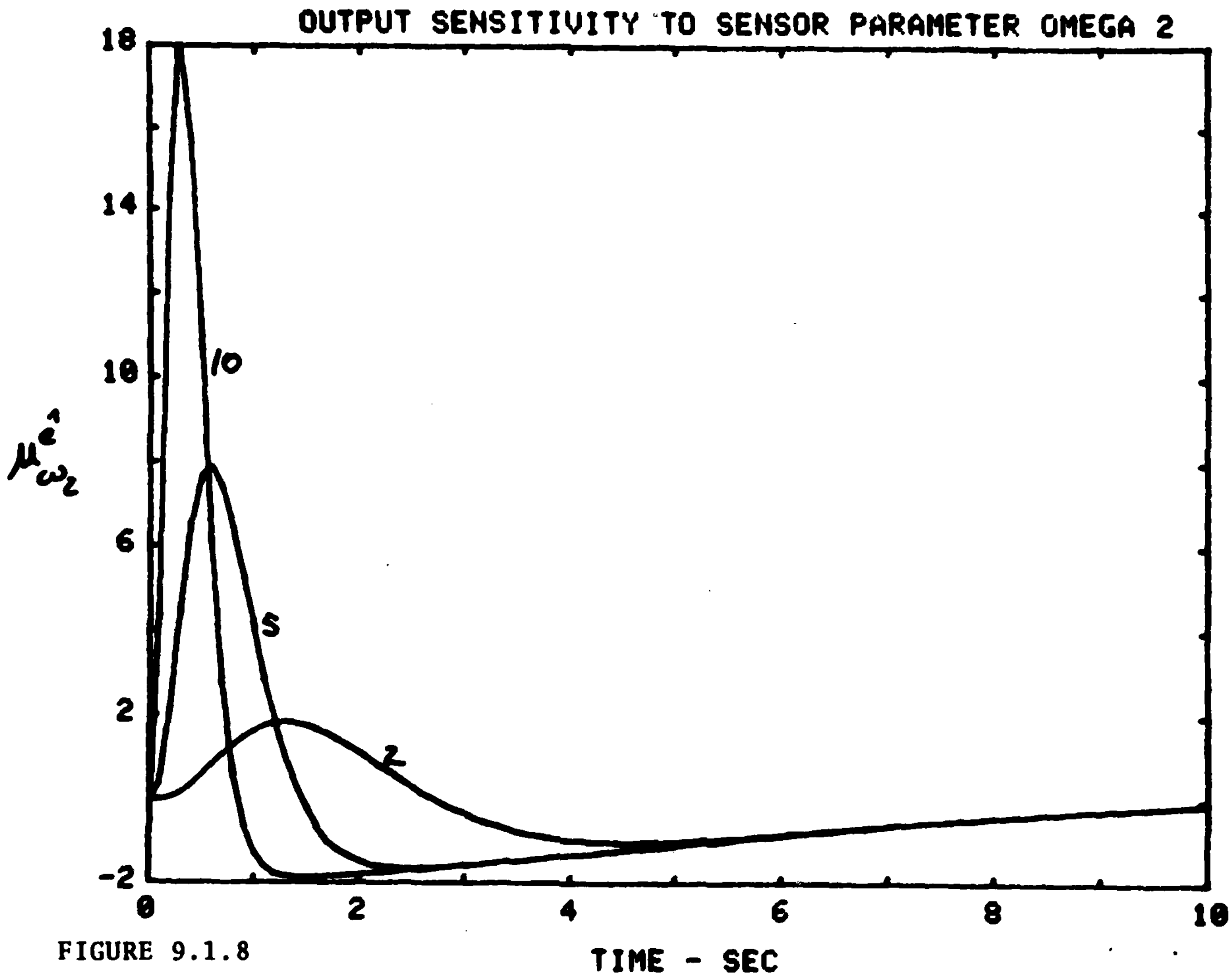

OUTPUT SENSITIVITY TO SENSOR PARAMETER OMEGA 1
1.2
0.8
0.4
0
-0.4
-0.8
0
4
8
12
16
20
TIME - SEC
10
5
2
$\mu^{\hat{e}}_{\omega_1}$

FIGURE 9.1.6



OUTPUT SENSITIVITY TO SENSOR PARAMETER ZETA 2
0.7
0.5
0.3
0.1
-0.1
-0.3
$\mu^{\hat{e}}_{\delta_2}$
10
5
2
0
0.4
0.8
1.2
1.6
2
TIME - SEC

FIGURE 9.1.7

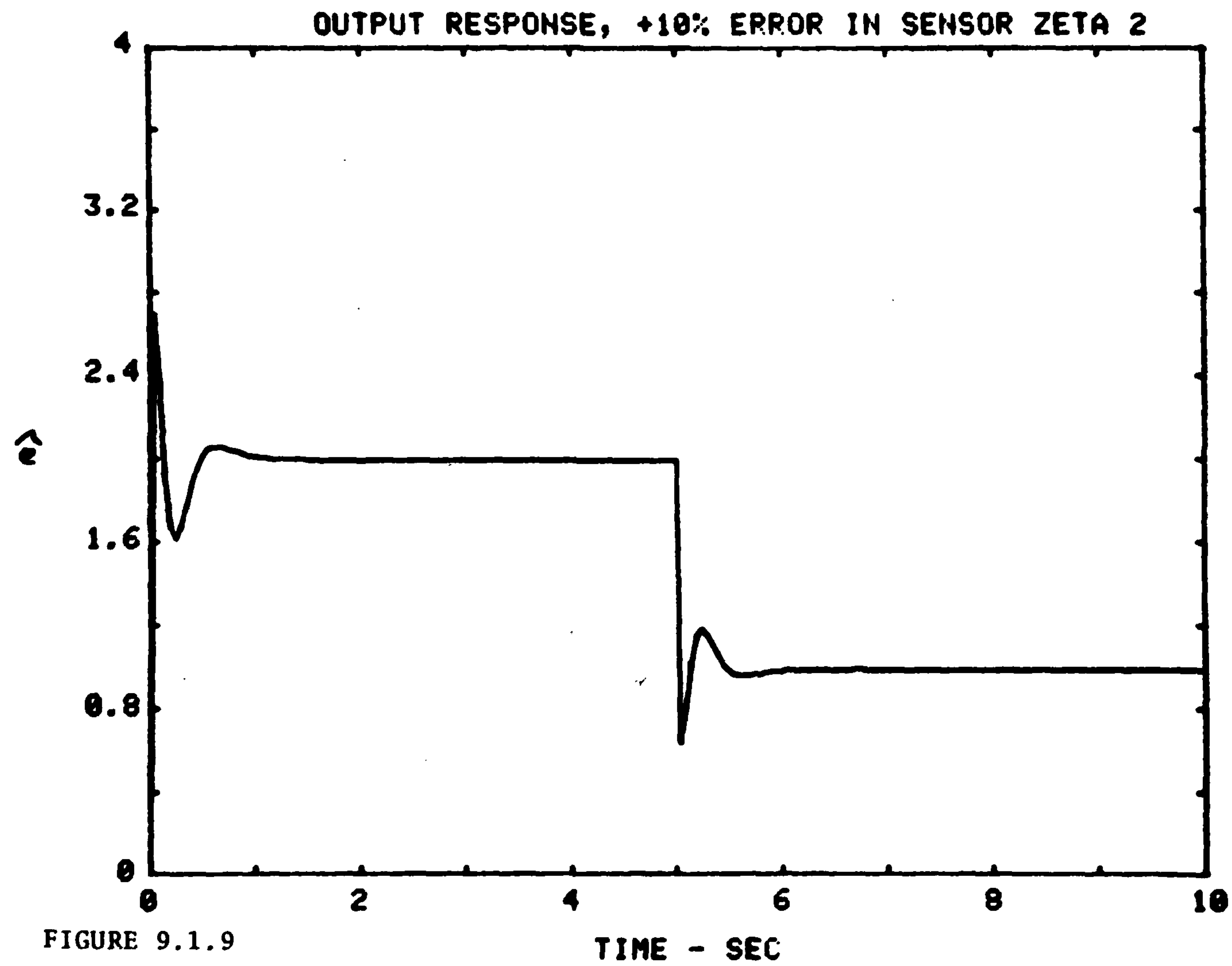


OUTPUT SENSITIVITY TO SENSOR PARAMETER OMEGA 2
18
14
10
6
2
-2
0
2
4
6
8
10
TIME - SEC
$\mu^{\hat{e}}_{\omega_2}$
10
5
2

FIGURE 9.1.8

generally always the case, the transient sensitivity increases as the observer gains $\underline{K}$ increase. However, there is no steady state deviation in $\hat{e}$ due to parameter error.

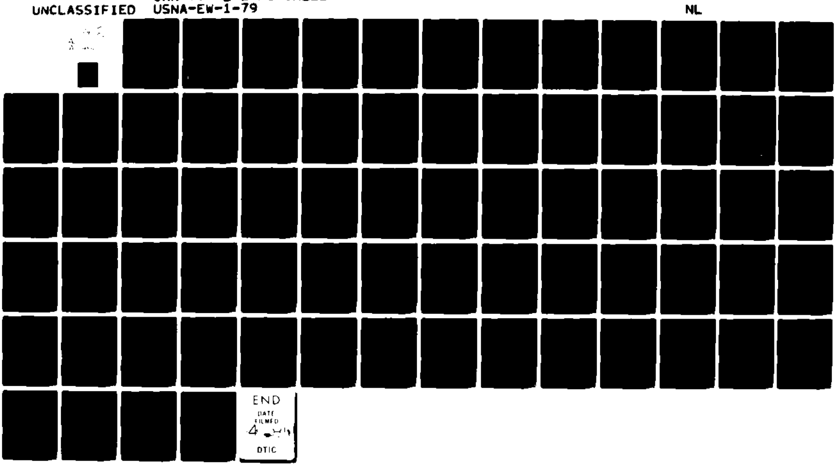
Figure 9.1.9, 10 and 11 illustrate the output of $\hat{e}$, (observer roots at -10) for $\dot{\theta}$ equal to a step of magnitude two, which drops to a magnitude of one at t = 5 sec. Transients at the steps increase, but again, the basic tracking ability remains.

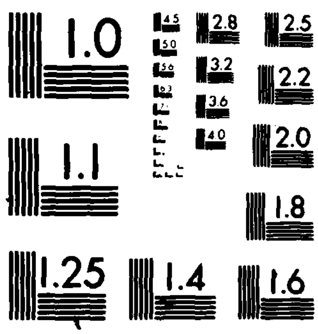


OUTPUT RESPONSE, +10% ERROR IN SENSOR ZETA 2
4
3.2
2.4
1.6
0.8
0
0
2
4
6
8
10
TIME - SEC

FIGURE 9.1.9

AD-A096 250 NAVAL ACADEMY ANNAPOLIS MD DIV OF ENGINEERING AND WEAPONS F/G 9/3
SENSITIVITY ANALYSIS OF A RATE SENSOR WITH OBSERVER.(U)
JAN 79 E E MITCHELL
UNCLASSIFIED USNA-EW-1-79 NL





MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

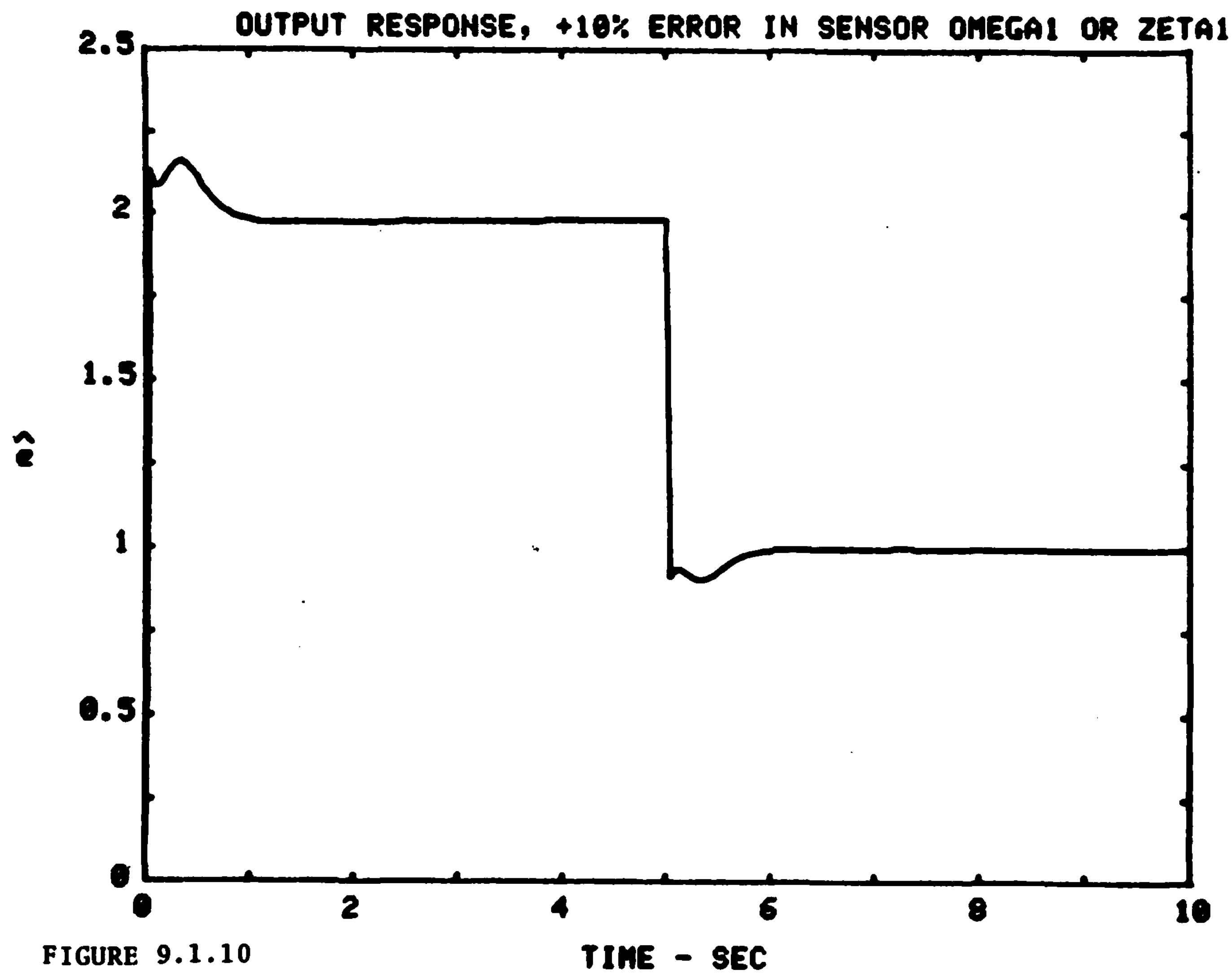
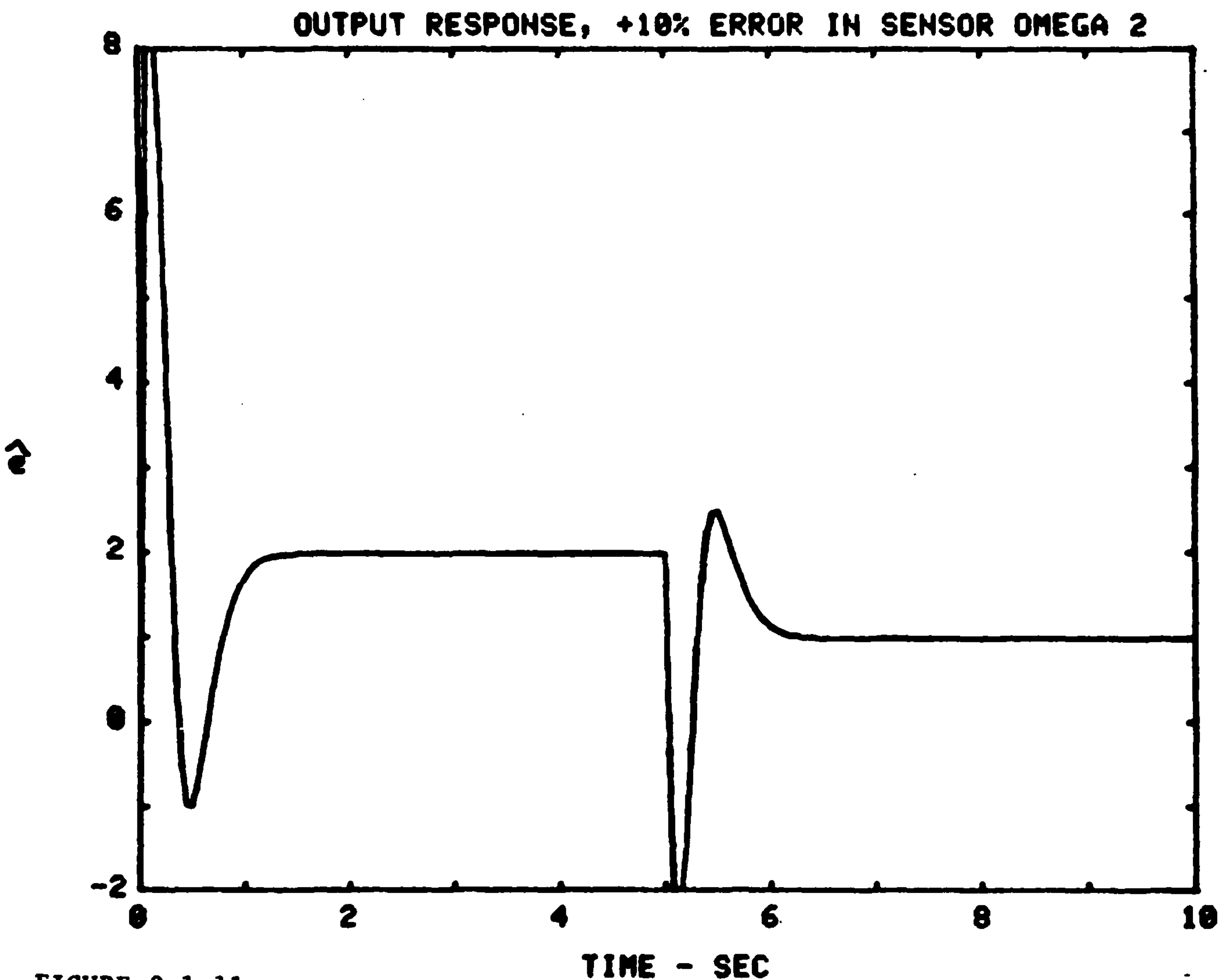

OUTPUT RESPONSE, +10% ERROR IN SENSOR OMEGA1 OR ZETA1
2.5
2
1.5
1
0.5
0
0
2
4
6
8
10
TIME - SEC

FIGURE 9.1.10


OUTPUT RESPONSE, +10% ERROR IN SENSOR OMEGA 2
8
6
4
2
0
-2
0
2
4
6
8
10
TIME - SEC
FIGURE 9.1.11

10.0 OUTPUT SENSITIVITY WITH RESPECT TO PARAMETERS OF THE OBSERVER

10.1 Sensitivity Equations

From Figure 2.2.1(b) the rate sensor equation is

$$e_o = C_4(sI-A)^{-1}B\theta \tag{10.1.1}$$

and the observer output is

$$\hat{e} = M(sI-F)^{-1}(B\theta + \underline{K}e_o) \tag{10.1.2}$$

The parameters of concern are those of the F matrix in (10.1.2). These are the four observer parameters δ_1, ω_1, δ_2 and ω_2. As in the last section, let p represent a general parameter. Then from (10.1.2)

$$\frac{\partial\hat{e}}{\partial p} = M(sI-F)^{-1}\,\frac{\partial F}{\partial p}\,(sI-F)^{-1}(B\theta + \underline{K}e_o) \tag{10.1.3}$$

Using equation (10.1.1) the terms

$$\begin{aligned}(sI-F)^{-1}(B\theta + \underline{K}e_o) &= [(sI-F)^{-1} + (sI-F)^{-1}\underline{K}C_4(sI-A)^{-1}]B\theta \\ &= (sI-A)^{-1}B\theta \end{aligned} \tag{10.1.4}$$

by use of the theorem (2.3.13). Hence (10.1.3) becomes

$$\frac{\partial\hat{e}}{\partial p} = M(sI-F)^{-1}\,\frac{\partial F}{\partial p}\,(sI-A)^{-1}B\theta \tag{10.1.5}$$

The F matrix is

$$F = \begin{bmatrix} 0 & 1 & 0 & -K_1 \\ 0 & 0 & 1 & -K_2 \\ 0 & 0 & 0 & 1-K_3 \\ -b & -c & -d & -(e+K_4) \end{bmatrix}$$

and since b, c, d, and e are functions of the parameters,

$$\frac{\partial F}{\partial p} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{\partial b}{\partial p} & -\frac{\partial c}{\partial p} & -\frac{\partial d}{\partial p} & -\frac{\partial e}{\partial p} \end{bmatrix} \qquad (10.1.6)$$

Comparing (10.15) to (9.1.5) it is seen that $\partial F/\partial p = \partial A/\partial p$, consequently, from equation (9.1.10), one may write

$$\frac{\partial F}{\partial p}(sI-A)^{-1}B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\frac{\partial}{\partial p}\Delta_A \end{bmatrix} \frac{a}{\Delta_A} \qquad (10.1.7)$$

In a like manner, from equation (9.1.12) one finds that

$$M(sI-F)^{-1} = \frac{1}{\Delta_F}\left[\Sigma m_i f_{i1} \;\; \Sigma m_i f_{i2} \;\; \Sigma m_i f_{i3} \;\; \Sigma m_i f_{i4}\right]_{i = 2\text{-}4} \qquad (10.1.8)$$

Combining equations (10.1.7) and (10.1.8), (10.1.5) becomes

$$\frac{\partial \hat{e}}{\partial p} = \frac{a}{\Delta_F \Delta_A}\left(-\frac{\partial}{\partial p}\Delta_A\right)\left(\sum_{i=2}^{4} m_i f_{i4}\right)\theta \qquad (10.1.9)$$

and since $\mu_p^{\hat{e}} = p\partial\hat{e}/\partial p$, we get

$$\mu_p^{\hat{e}} = (p \frac{\partial}{\partial p} \Delta_A) \frac{-a}{s\Delta_F\Delta_A} (\sum_2^4 m_i f_{i4})\theta \tag{10.1.10}$$

Note that (10.1.10) has θ as its input. The term $p\partial\Delta_A/\partial p$ is identically the same as found in the last section. The results are repeated below.

For δ_1;

$$p\frac{\partial}{\partial p} \Delta_A = 2\delta_1 \omega_1 s\Delta_{A2} \tag{10.1.11}$$

for ω_1;

$$p\frac{\partial}{\partial p} \Delta_A = (2\delta_1 \omega_1 s + 2\omega_1^2) \Delta_{A2} \tag{10.1.12}$$

for δ_2;

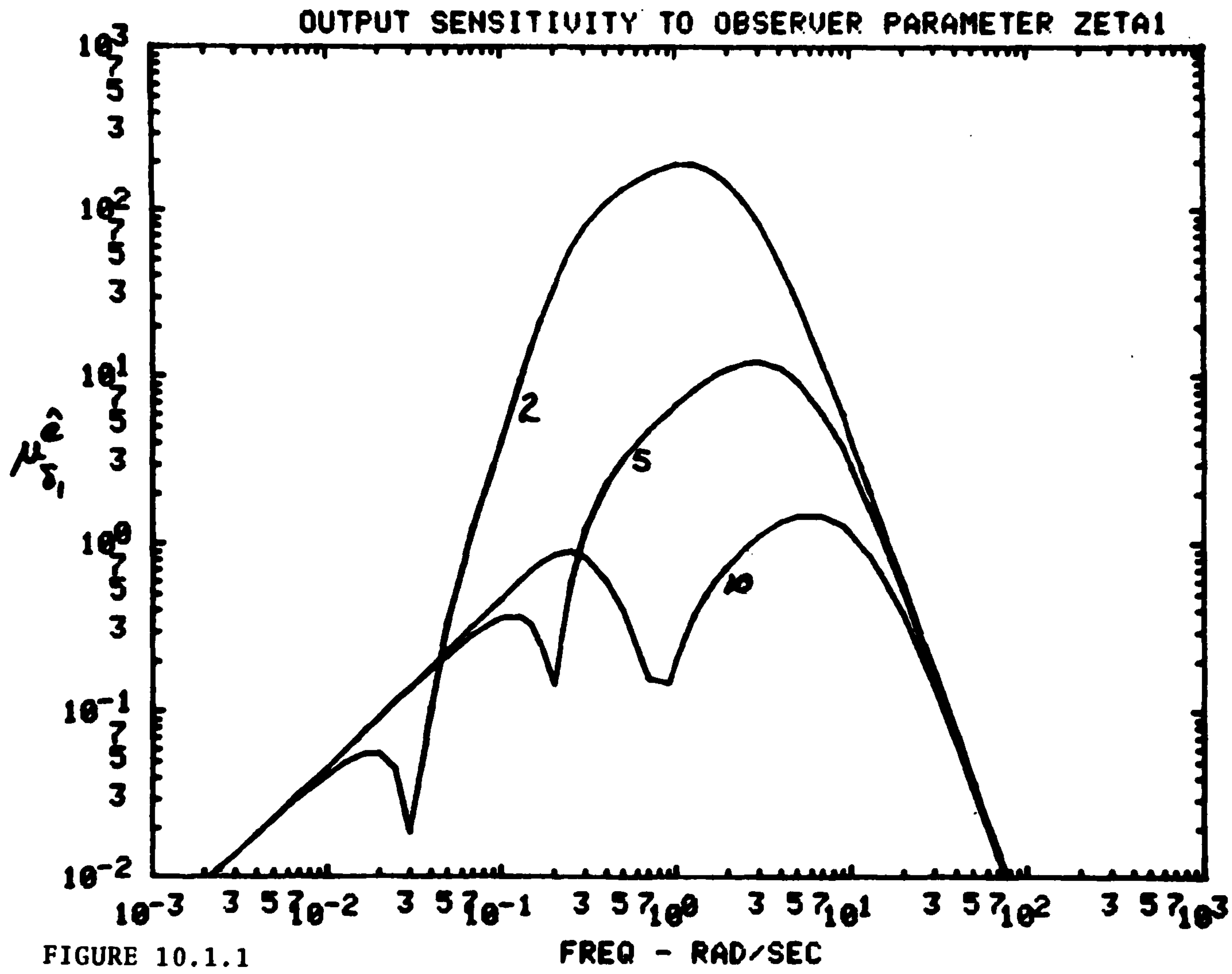
$$p\frac{\partial}{\partial p} \Delta_A = 2\delta_2\omega_2 s\Delta_{A1} \tag{10.1.13}$$

for ω_2;

$$p\frac{\partial}{\partial p} \Delta A = (2\delta_2\omega_2 s + 2\omega_2^2) \Delta_{A1} \tag{10.1.14}$$

Figures 10.1.1-4 illustrate the sensitivity of $\hat{e}$ to the observer parameters δ_1, ω_1, δ_2 and ω_2 respectively.

The summation term on the right side of (10.1.10) was expanded in equation (6.3.4). The term was shown to reduce to unity under steady state conditions in equation (6.3.5), assuming a unit step input at $\hat{\theta}$.



OUTPUT SENSITIVITY TO OBSERVER PARAMETER ZETA1
$\mu^{\hat{e}}_{\delta_1}$
2
5
10
FREQ - RAD/SEC

FIGURE 10.1.1

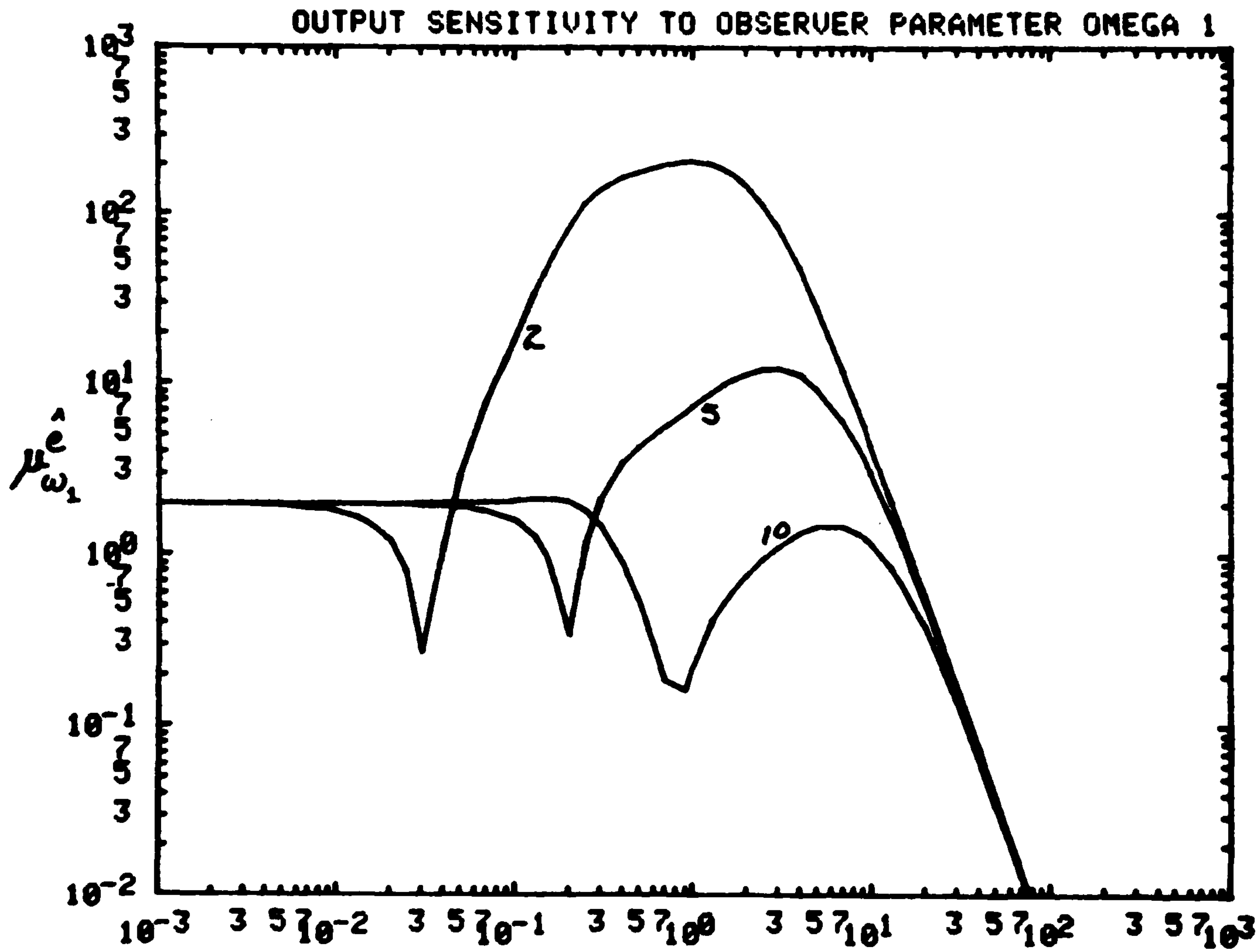

OUTPUT SENSITIVITY TO OBSERVER PARAMETER OMEGA 1
$\hat{\mu}^{e}_{\omega_1}$
FREQ - RAD/SEC
2
5
10

FIGURE 10.1.2

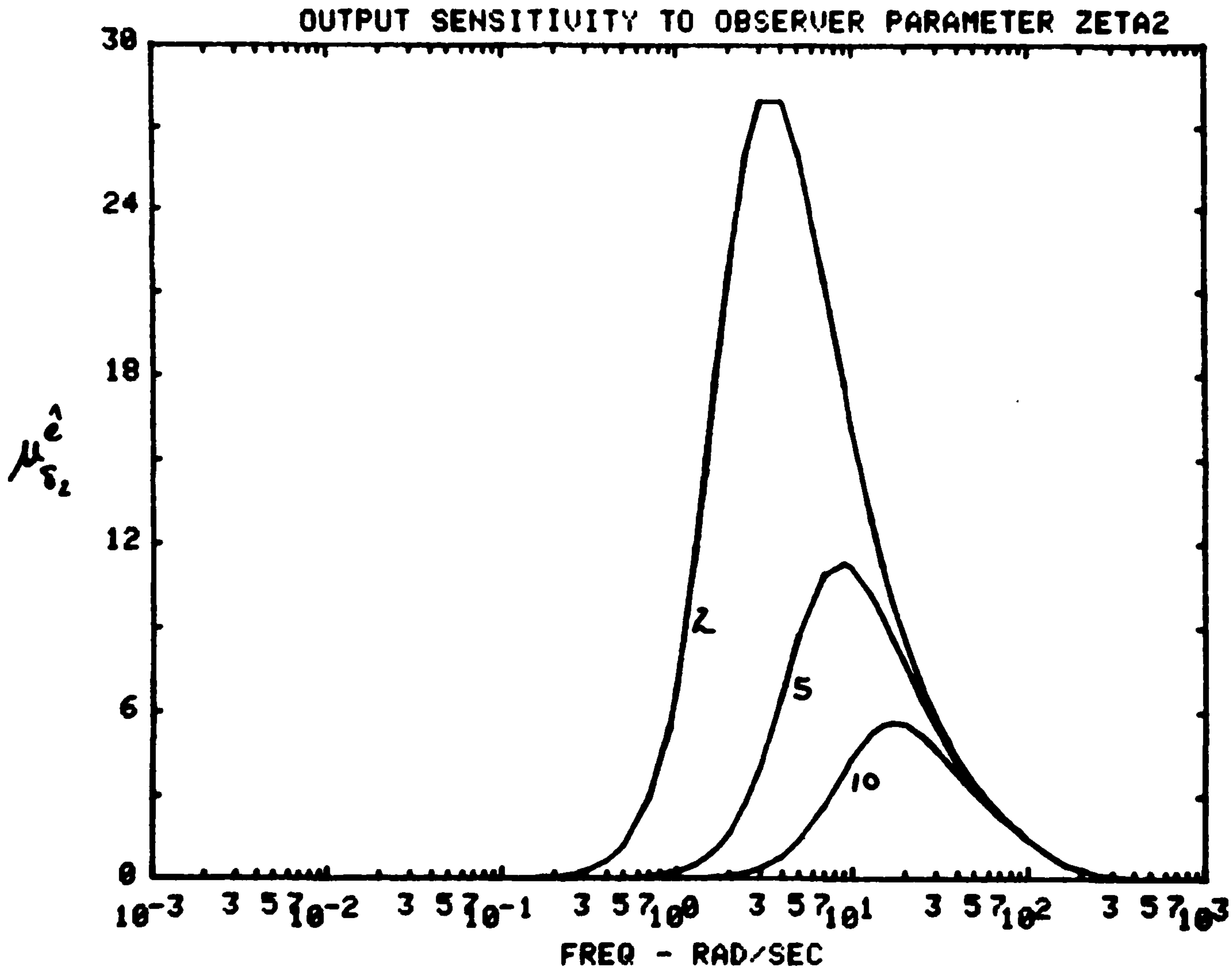
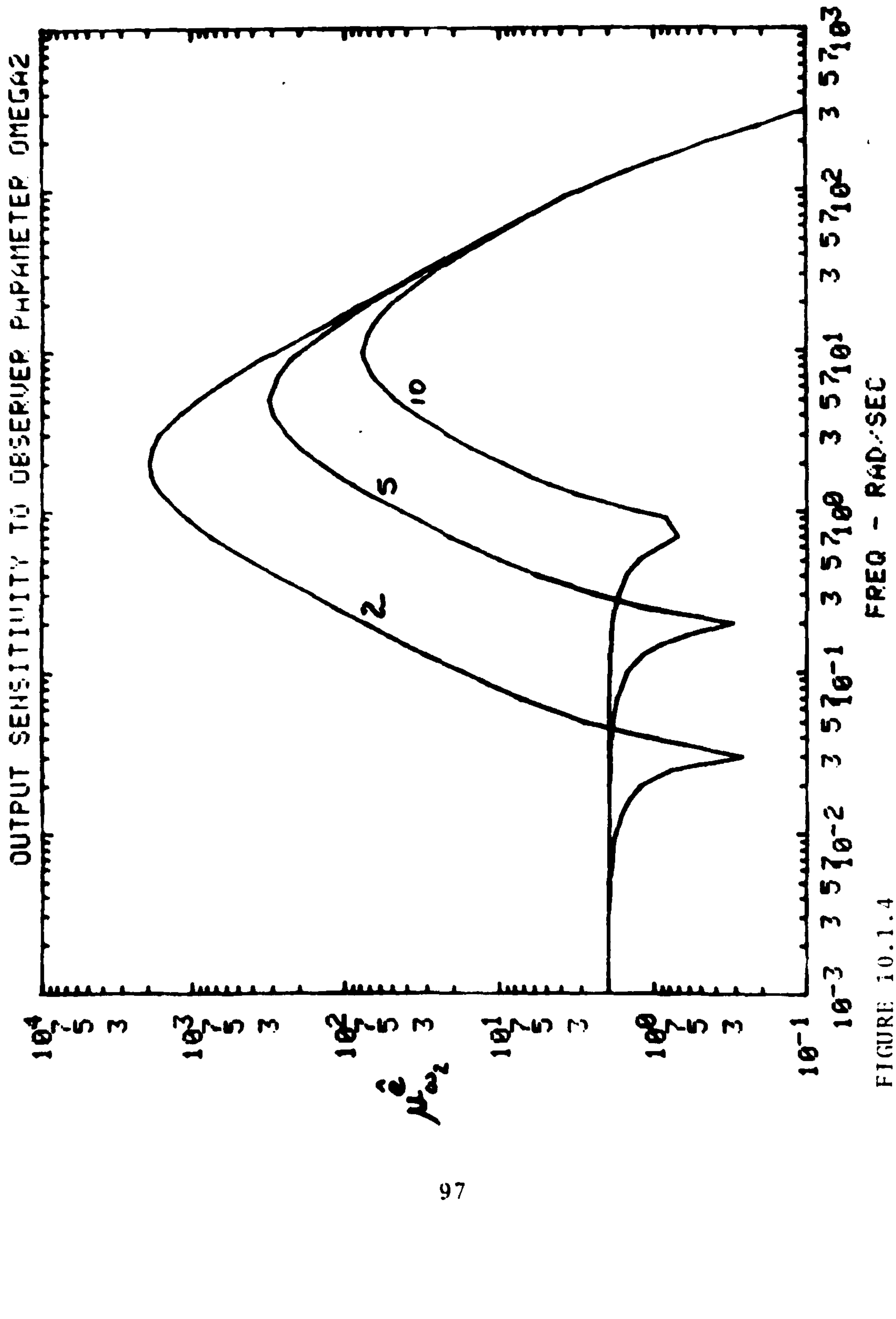

OUTPUT SENSITIVITY TO OBSERVER PARAMETER ZETA2
30
24
18
12
6
0
10-3 3 5 7 10-2 3 5 7 10-1 3 5 7 10^0 3 5 7 10^1 3 5 7 10^2 3 5 7 10^3
FREQ - RAD/SEC
2
5
10

FIGURE 10.1.3



OUTPUT SENSITIVITY TO OBSERVER PARAMETER OMEGA2
FREQ - RAD/SEC
2
5
10

FIGURE 10.1.4

Therefore, assuming a unit step input at $\dot{\theta}$, under steady state conditions, equation (10.1.15) becomes

$$\mu_p^{\hat{e}}\Big|_{ss} = \lim_{s \to o} s \left[\frac{1}{s\Delta_A} \; p \frac{\partial}{\partial p} \Delta_A \right]$$

When considering either of the δ terms, equation (10.1.11) or (10.1.13) applies, and $\mu_\delta^{\hat{e}}|_{ss} = 0$. This is shown in Figures 10.1.1 and 10.1.3.

When considering either of the ω terms, equation (10.1.12) or (10.1.14) applies, Using (10.1.12) gives

$$\mu_{\omega_1}^{\hat{e}}\Big|_{ss} = \frac{2\omega_1^2\omega_2^2}{\omega_1^2\omega_2^2} = 2$$

which also holds for ω_2. This is seen in Figure 10.1.2 and 10.1.3.

Figures 10.1.5-8 illustrate the time response of the parameter sensitivity equation (10.1.10) for δ_1, ω_1, δ_2 and ω_2 respectively.

Figure 10.1.9 and 10 show the output response for +10% errors in δ_2 and ω_2 respectively, (observer roots at -10). For comparison, the response of the ideal system is also plotted. The input , $\dot{\theta}$, was aunit step until t = 3, at which time it went to a magnitude of 2, and then returned to unity at t = 5. In both cases the basic tracking ability remains, however, the parameter errors cause some transients.

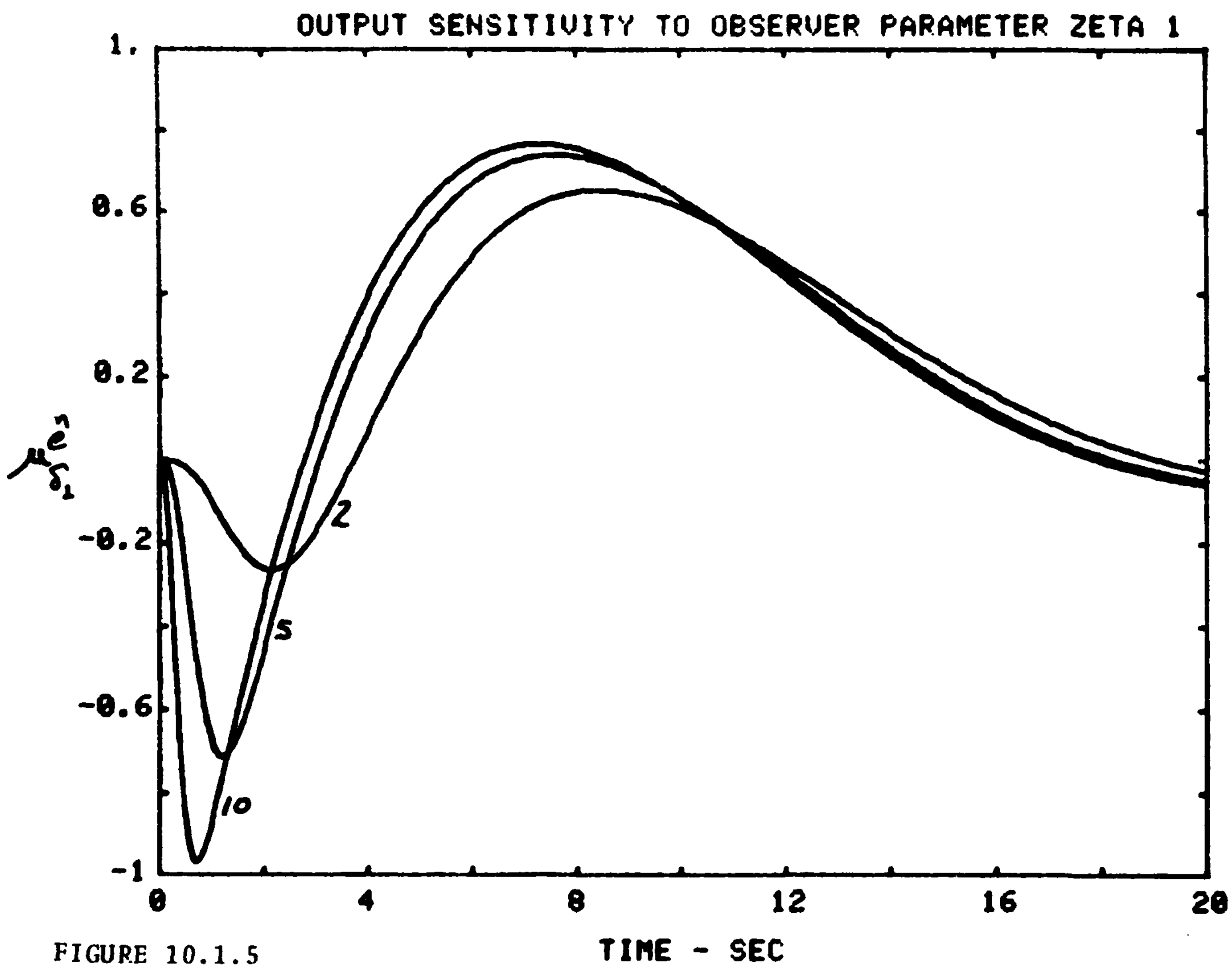
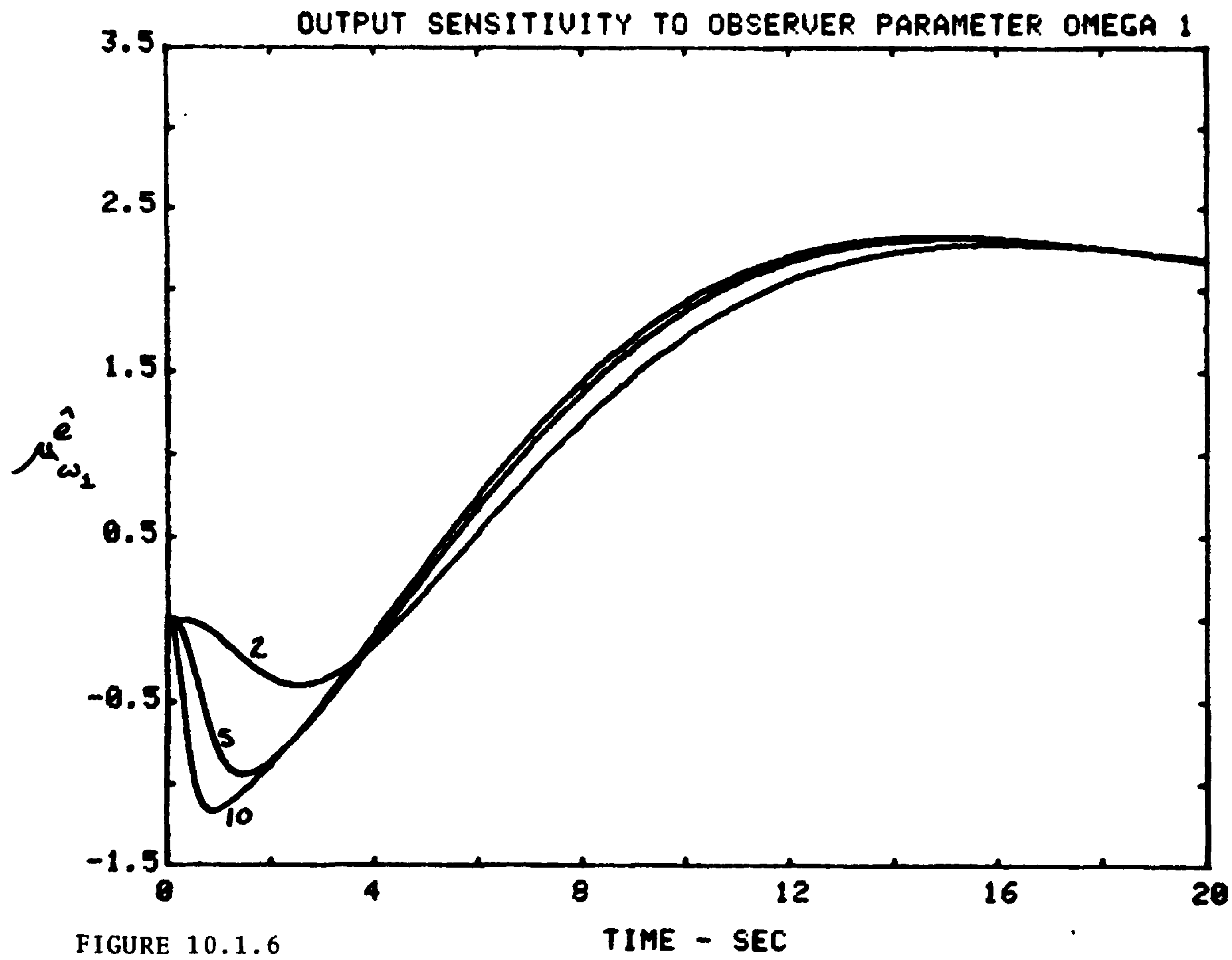

OUTPUT SENSITIVITY TO OBSERVER PARAMETER ZETA 1
1.
0.6
0.2
-0.2
-0.6
-1
0
4
8
12
16
20
TIME - SEC
2
5
10

FIGURE 10.1.5



OUTPUT SENSITIVITY TO OBSERVER PARAMETER OMEGA 1
3.5
2.5
1.5
0.5
-0.5
-1.5
0
4
8
12
16
20
TIME - SEC
2
5
10

FIGURE 10.1.6

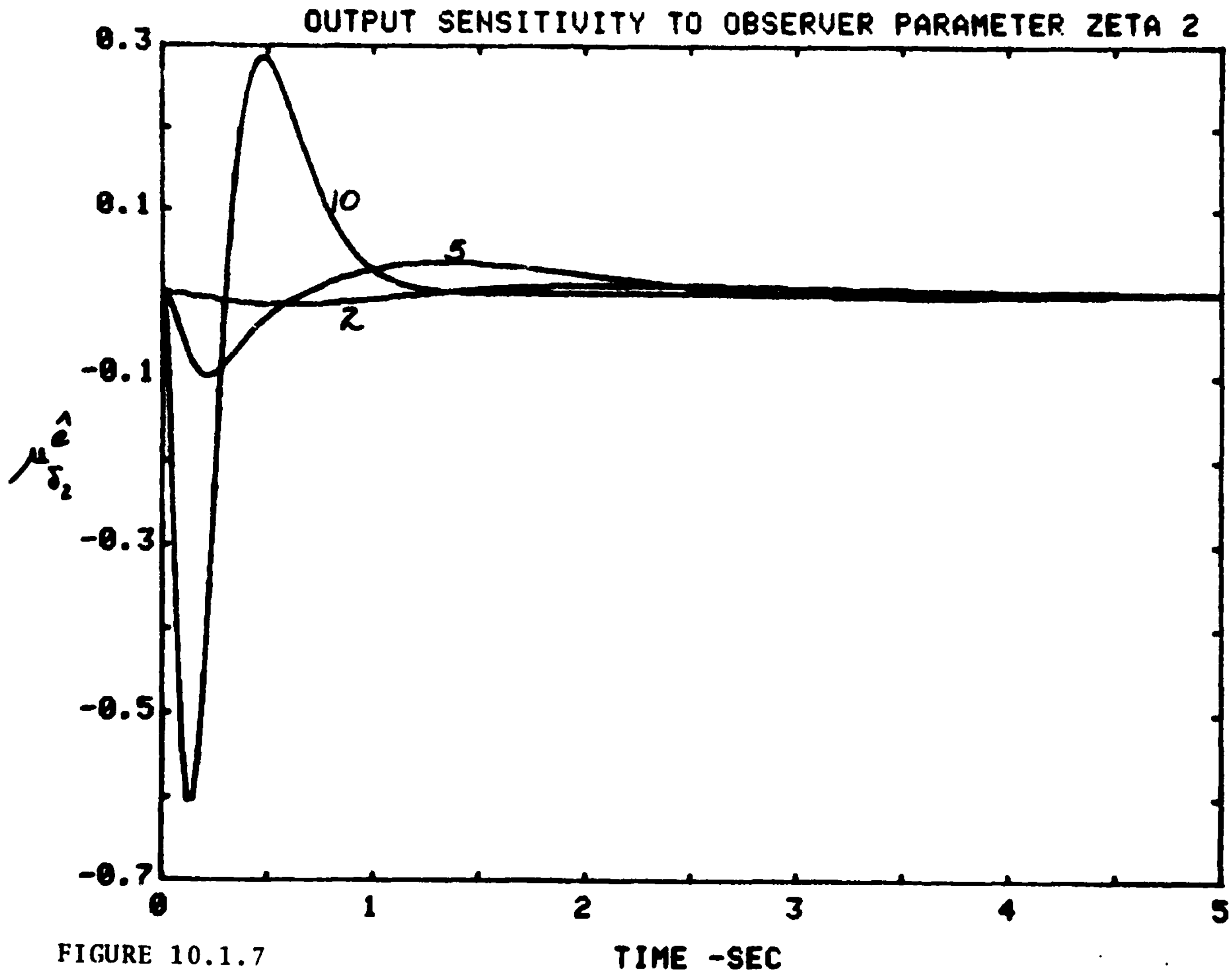

OUTPUT SENSITIVITY TO OBSERVER PARAMETER ZETA 2
0.3
0.1
-0.1
-0.3
-0.5
-0.7
0
1
2
3
4
5
10
5
2
TIME -SEC

FIGURE 10.1.7

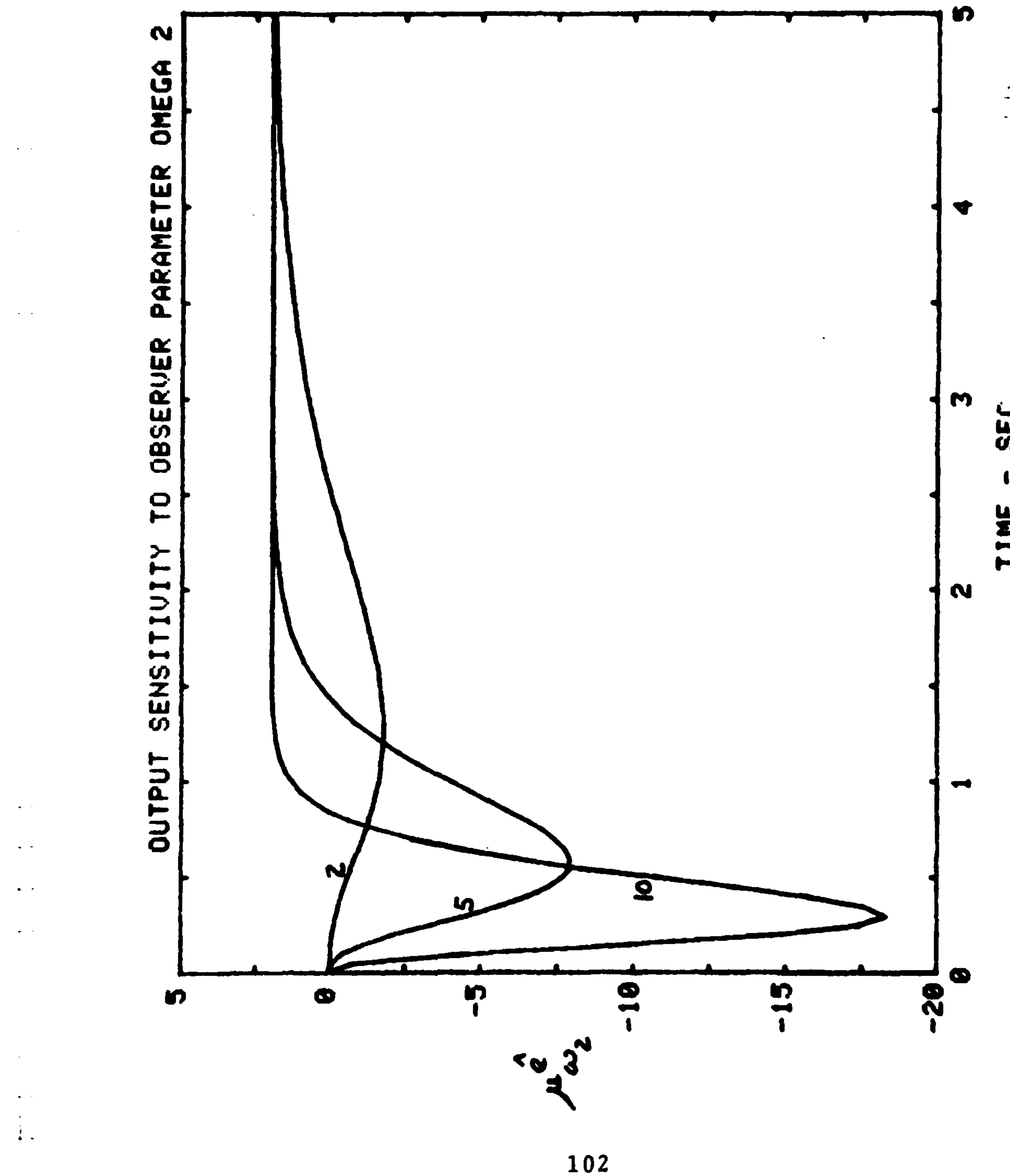

OUTPUT SENSITIVITY TO OBSERVER PARAMETER OMEGA 2
TIME - SEC
$\mu_{\omega_2}^{\hat{e}}$
5
0
-5
-10
-15
-20
0
1
2
3
4
5
2
5
10

FIGURE 10.1.8

Figure 10.1.11 is a run of $\hat{e}$ with +5% error in ω_1, and is a very poor response. Additional runs for δ_1 were unstable for even +5% error in δ_1. On the other hand, Figures 10.1.12 and 13 were runs made with -10% error in ω_1 and δ_1 respectively. Since these are low frequency terms, the transients don't decay rapidly, but the response is well behaved.

If the input $\dot{\theta}$ is a unit ramp, then

$$\mu_{\omega}^{\hat{e}} \Big|_{ss} = \lim_{s \to o} \frac{2}{s} \to \infty$$

This means that the error grows under conditions of long term constant acceleration. This is the same result as discussed in Section 5.

In a similar manner, if $\dot{\theta}$ is a unit ramp, then

$$\mu_{\delta_1}^{\hat{e}} \Big|_{ss} = \frac{2\delta_1}{\omega_1}$$

and

$$\mu_{\delta_2}^{\hat{e}} \Big|_{ss} = \frac{2\delta_2}{\omega_2}$$

which means a small, constant error under constant acceleration.

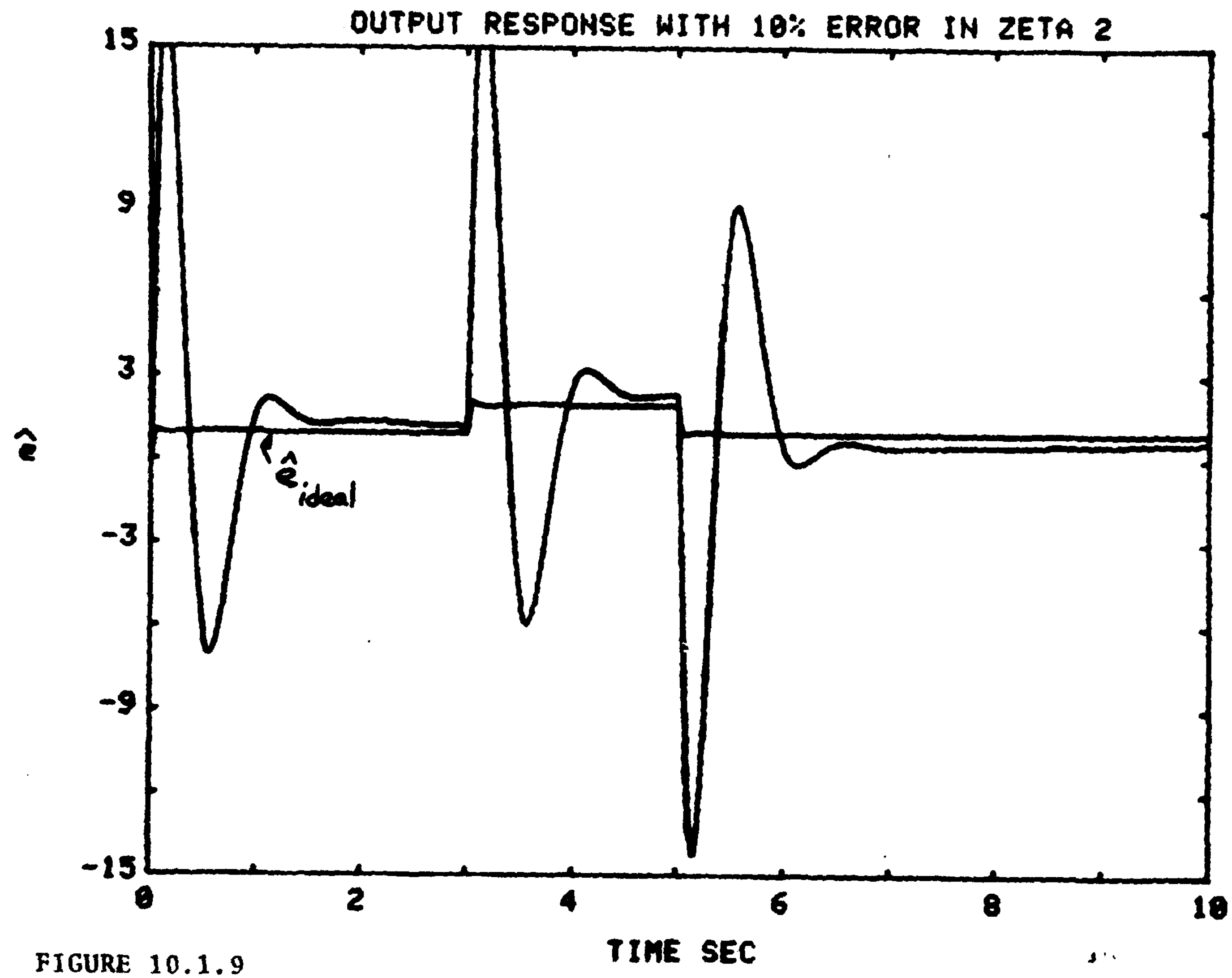

OUTPUT RESPONSE WITH 10% ERROR IN ZETA 2
15
9
3
-3
-9
-15
0
2
4
6
8
10
TIME SEC
ê
ê ideal

FIGURE 10.1.9

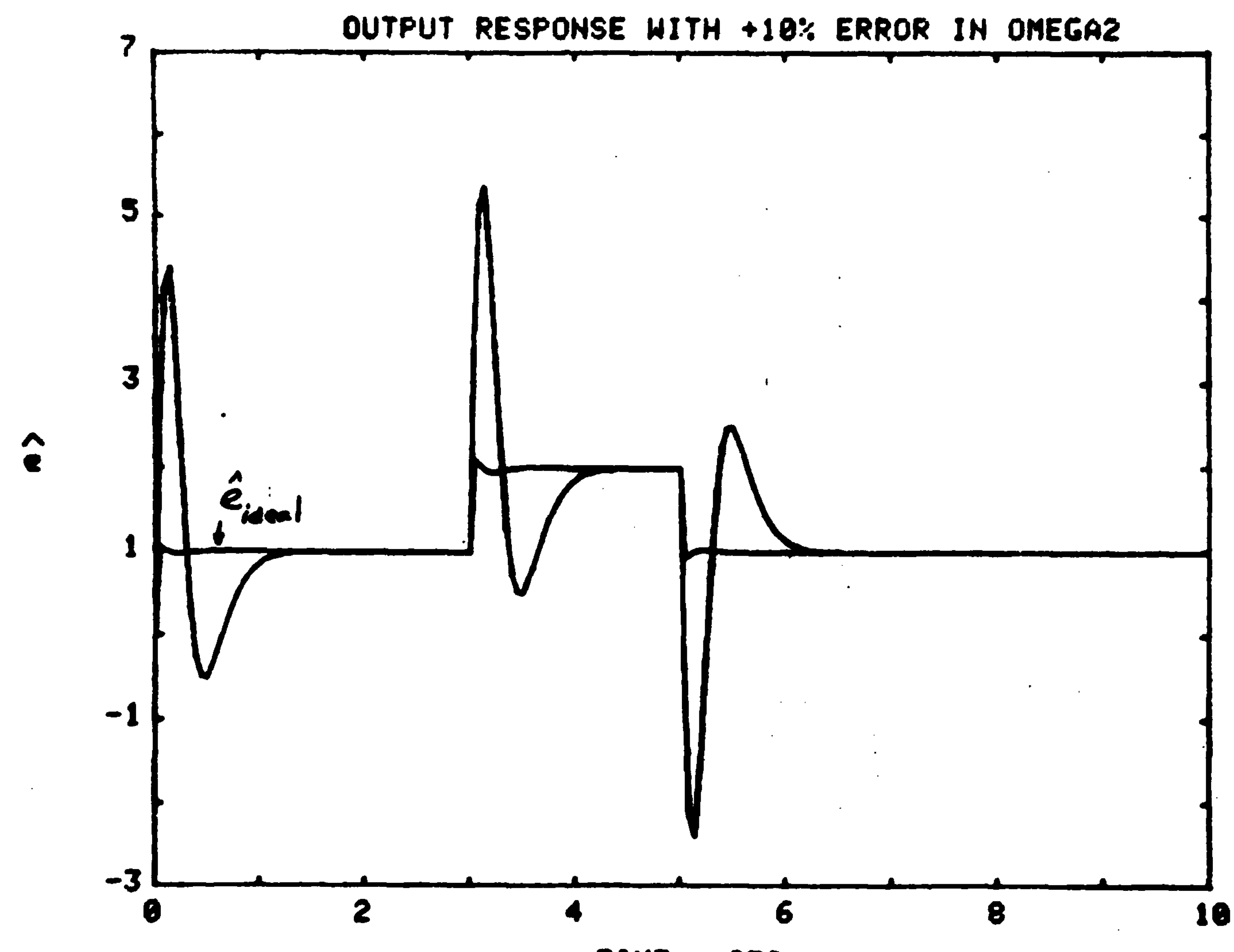


FIGURE 10.1.10

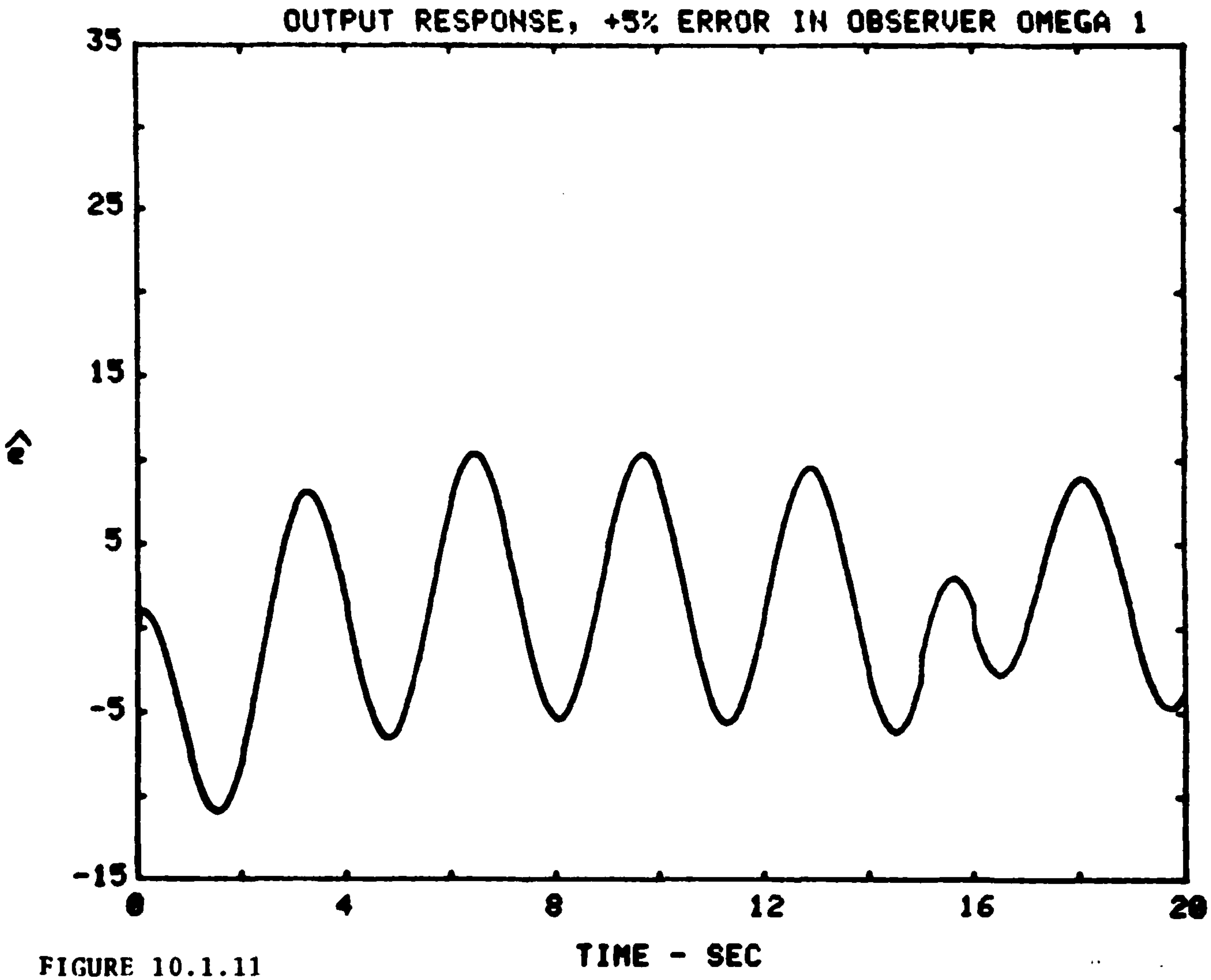


FIGURE 10.1.11

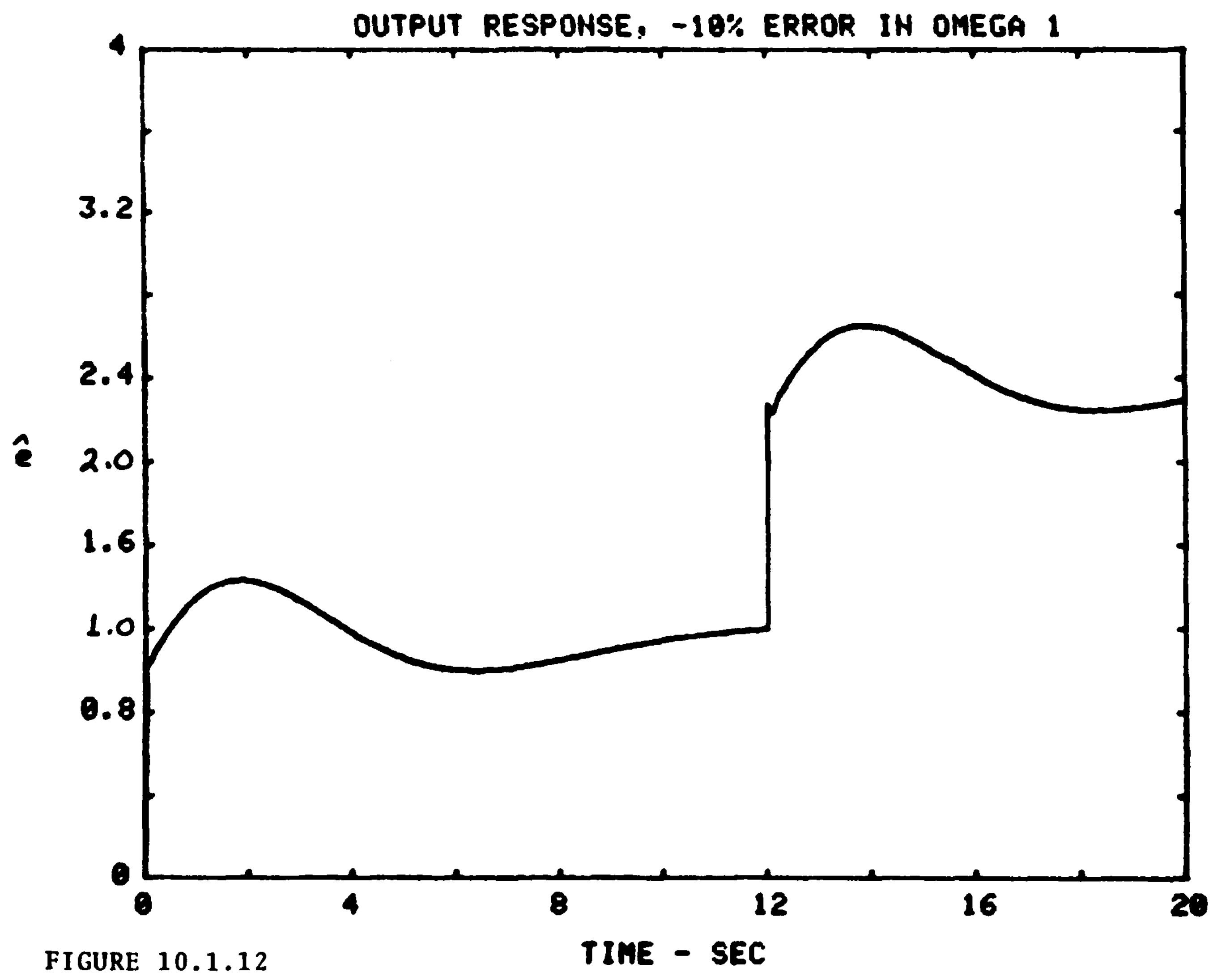


FIGURE 10.1.12

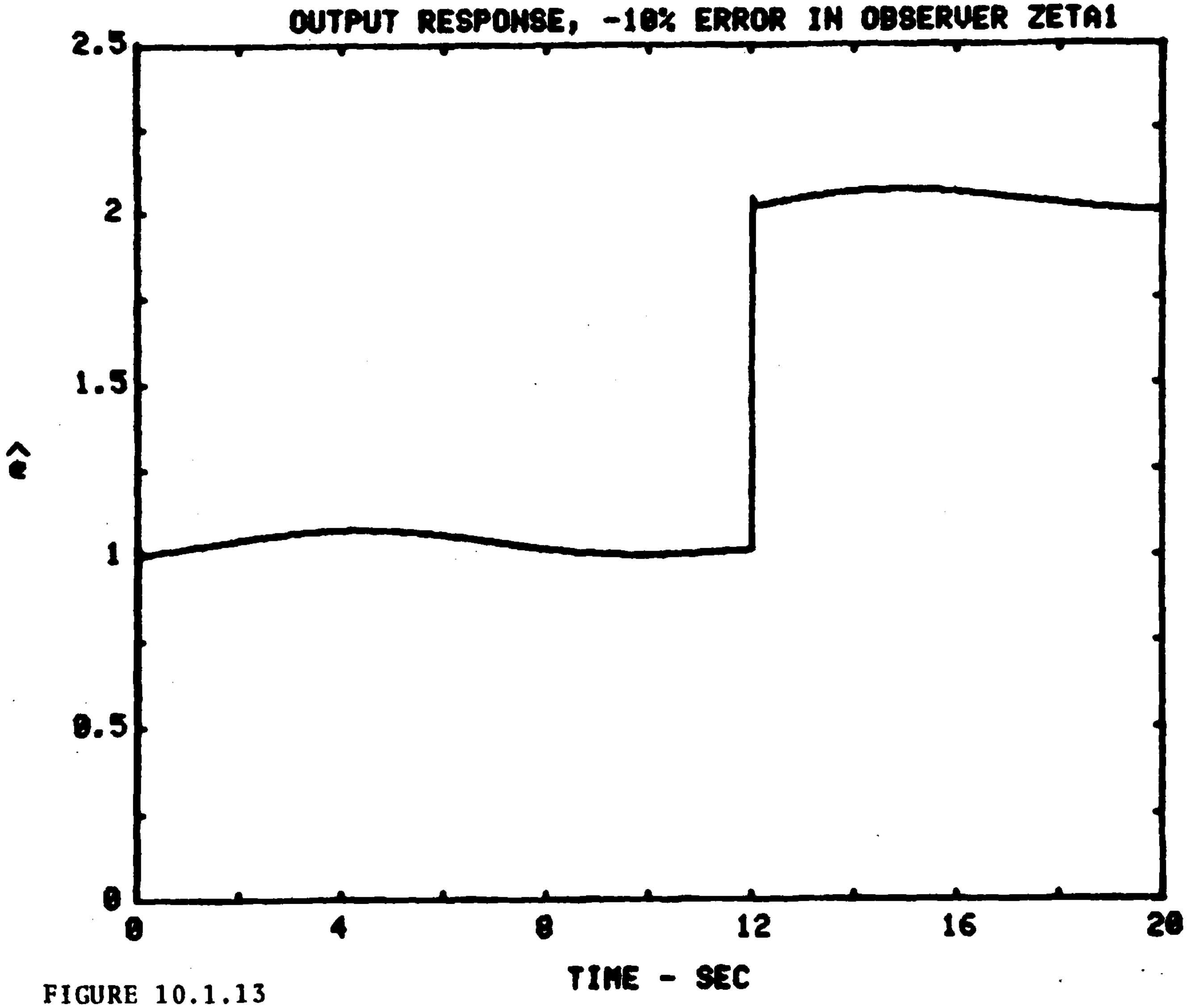


FIGURE 10.1.13

11.0 SUMMARY

In general a sensitivity study involves a large amount of data, and this one is no different. A condensation based on effects is attempted below.

When the velocity, $\dot{\theta}$ suffers a step change, errors in the observer parameters ω_1, ω_2 and gain a; the output matrix element m_2; and steady, offset errors in θ the generated signal, affect the steady state output of the observer, $\hat{e}$.

When the velocity $\dot{\theta}$ is a ramp, constant errors in the above mentioned parameters tends to cause ever increasing errors in the output $\hat{e}$. In addition, the output matrix element m_3 and observer parameters δ_1 and δ_2 affect the steady state output of the observer. These would appear to be "second order" effects and will most likely not appear in the actual aircraft.

After initial condition transients have died and the observer has reached steady state, the following parameters affect, to some extent, the performance of the observer while tracking step changes in $\dot{\theta}$.

Observer gain a

Sensor parameters δ_2 and ω_2

Observer parameters δ_1, ω_1, δ_2 and ω_2

Noise on the generated signal θ

The major problem appears to be with the sensor parameters ω_1 and δ_1. If these are in error on the low side an acceptable response results. If they are in error on the high side, the solution is unstable.

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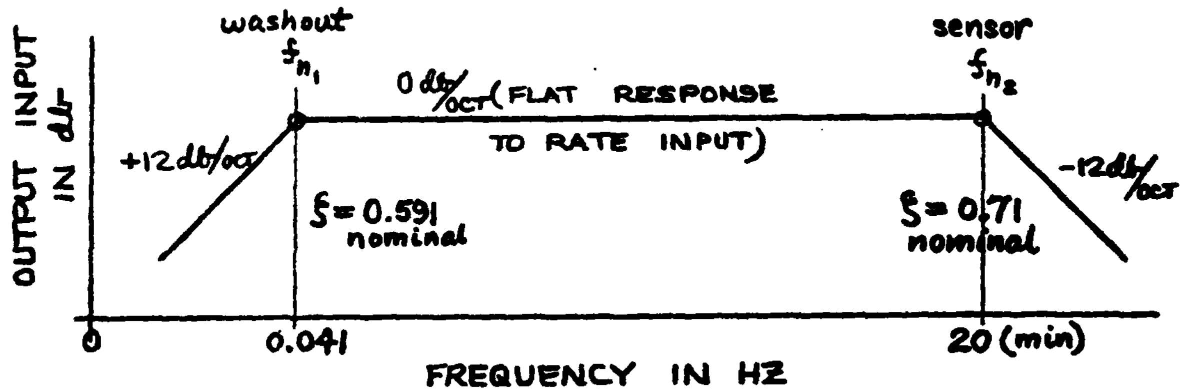
SYSTRON-DONNER MODEL 8160 RATE SENSOR TRANSFER FUNCTION

P/N 8160-120-P15

$$\frac{E_0}{\theta}(s) = 0.075\left(\frac{S^2}{S^2+0.303S+0.0656}\right)\left(\frac{15600}{S^2+175S+15600}\right) \text{VDC/deg/sec}$$

CORRESPONDING AMPLITUDE RESPONSE:



INERTIAL DIVISION
SYSTRON DONNER
CORPORATION
CONCORD, CALIFORNIA

8-2-76

APPENDIX B. COMPUTER PROGRAMS FOR FREQUENCY RESPONSE OF SENSITIVITY EQUATIONS

A sequence of main programs, all essentially the same, were written to generate the polynomials. The main programs then called the subroutine SENSUB to generate the frequency response curves. The algorithm used in SENSUB is explained in reference [17].

The programs are as follows:

SENSE - computes the frequency response of $\hat{e}$.

Compute the frequency response of $\hat{e}$ with respect to:

SENSEM - the m_i of the output matrix

SENSEAO - the observer gain

SENSEAS - the rate sensor gain

SENSEZ1 - δ_1 of rate sensor

SENSEW_1 - W_1 of rate sensor

SENSEZ2 - δ_2 of rate sensor

SENSEK - observer gains

SENSEW10 - W_1 of observer

SENSEZ10 - δ_1 of observer

SENSEW20 - $_2$ of observer

SENSEZ20 - δ_2 of observer

SENSUB - frequency response subroutine

POLYPAK - polynomial subroutines

The programs SENSEZ1, SENSEω1, SENSEZ2 and SENSEω2 are the same except for the polynomial forms. Hence, SENSEZ1

is listed, but only the altered sections are listed for the other three programs. The same is true for SENSEK, SENSEω10, SENSEω20, SENSEZ10 and SENSEZ20.

SENSE

```
100***  MAIN PROGRAM TO COMPUTE THE FREQUENCY
120***     RESPONSE OF E(S)
140***
160***
180        COMMON GN,GD,IGN,IGD,WN,WI,IGNP,IGDP,CV
200        IMPLICIT DOUBLE PRECISION(A-H,O-Z)
220        DOUBLE PRECISION K1,K2,K3,K4,M
240        INTEGER IGN(20),IGD(20),WN
260        DIMENSION GN(20,10),GD(20,10)
265        DIMENSION F(3,4),H(3,7)
266        DIMENSION M(3),DELA1(3),DELA2(3),DELF(7)
280 CHARACTER CV(2,40)
281 CHARACTER*1 P1(40),P2(40)
285 DATA P2/3*1HA,1H*,1H*,1H@/
287 DATA P1/3*"A", "+","+",2*"A",2*"*","S",3*"A",2*"+",
288   &  "A","*","R","+","A","*","@"/
300 LIBRARY "SENSUB"
320 P2(4)="*"
340*
360*
380*
400        M(1)=.0656/.075
420        M(2)=.303/.075
440        M(3)=1./.075
460 PRINT,"M2,M3,M4",M
480 A=1170.
500 B=.0656*15600.
520 C=.303*15600. + .0656*175.
540 D=15600.+0.0656+.303*175.
560 E=175.303
580 PRINT,"A,B,C,D",A,B,C,D,E
600 PRINT,"INPUT,N"
610 INPUT,N
620 DO 400 IK=1,N
640 PRINT,"R COEFFS"
660 INPUT,R2,R3,R4,R5
680 PRINT,"R2,R3,R4,R5",R2,R3,R4,R5
700 K4 = R2-E
720 K3 = (B-R5)/B
740 K2 = (C*(R5/B)-R4)/B
760 K1 = (D*(R5/B)-C*K2-R3)/B
780 PRINT,"K1,K2,K3,K4",K1,K2,K3,K4
800*
820   IH2X=7
825 H(1,7)=K2
830 H(1,6) = K3 + K2*E
840 H(1,5) = K4 + K3*E + K2*D
850 H(1,4) = -K1*B
860*
880 H(2,7) = K3
```

SENSE (continued)

```
890 H(2,6) = K3*E+K4
900 H(2,5) = -K1*B - K2*C
910 H(2,4) = -K2*B
920*
940 H(3,7) = K4
950 H(3,6) = -(K1*B + K2*C + K3*D)
960 H(3,5) = -K2*B - K3*C
970 H(3,4) = -K3*B
980*
990*
1000   IF2X1=3
1010 F(1,3) = -K2
1020 F(1,2) = -(K3-1.)
1030*
1050 F(2,3) = -(K3-1.)
1060*
1079   IF2X2=4
1080 F(3,4)=1.
1090  IDELA=3
1091*
1093 DELA1(3)=1.
1094 DELA1(2)=.303
1095 DELA1(1)=.0656
1097*
1098 DELA2(3)=1.
1099 DELA2(2)=175.
1100 DELA2(1)=15600.
1102*
1104 IDELF=5
1105 DELF(5)=1
1106 DELF(4)=R2
1107 DELF(3)=R3
1108 DELF(2)=R4
1109 DELF(1)=R5
1110*
1120 DO 100 I=1,2
1130 100 IGN(I)=IF2X1
1131 IGN(3)=IF2X2
1135*
1140 DO 110 I=1,3
1150 DO 110 J=1,4
1160 110 GN(I,J) = M(I)*F(I,J)
1170*
1180 IGN(4)=IDELA
1190 IGN(5)=IDELA
1200 DO 115 I=1,3
1210 GN(4,I)=DELA1(I)
1220 115 GN(5,I)=DELA2(I)
1270*
1280 DO 120 I=1,3
```

SENSE (continued)

```
1290 IGN(5+I)=IH2X
1300 120 CONTINUE
1310*
1320 DO 130 I=1,3
1340 DO 130 J=4,IH2X
1350 130 GN(5+I,J) = M(I)*H(I,J)
1360*
1370 IGN(9)=1
1380 GN(9,1)=1
1390*
1400 IGN(10)=1
1410 GN(10,1)=1170.
1420*
1430*
1440*
1450 IGD(1)=IDELA
1460 IGD(2)=IDELA
1480 DO 140 J=1,3
1490 140GD(1,J)=DELA1(J)
1500 DO 150 J=1,3
1510 150 GD(2,J)=DELA2(J)
1520*
1530 IGD(3)=IDELF
1540 DO 160 I=1,5
1550 160 GD(3,I)=DELF(I)
1560*
1570*
1580 IGNP=10
1590 IGDP=3
1591 IGDP=4
1592 GD(4,2)=1
1593 IGD(4)=2
1600 P2(6)="A"
1601 P2(7)="*"
1602 P2(8)="@"
1615*
1620 DO 180 I=1,40
1630 CV(1,I)=P1(I)
1640 180 CV(2,I)=P2(I)
1645 PRINT,"P1",(P1(I),I=1,10)
1646 PRINT,"P1",(P1(I),I=11,20)
1647 PRINT,"P2",(P2(I),I=1,10)
1660 WN=7
1661 WI=.01
1670*
1680 CALL SENSUB
1685 400 CONTINUE
1690*
1700 END
```

SENSEM

```
100***  MAIN PROGRAM TO COMPUTE THE SENSITIVITY
120***    OF E WITH RESPECT TO M(I)
140***
160***
180       COMMON GN,GD,IGN,IGD,WN,WI,IGNP,IGDP,CV
185       COMMON ICNT
200       IMPLICIT DOUBLE PRECISION(A-H,O-Z)
220       DOUBLE PRECISION K(4),M
240       INTEGER IGN(20),IGD(20),WN
260       DIMENSION GN(20,10),GD(20,10)
280       DIMENSION F(4,4,4),IF(4,4)
300       DIMENSION M(4),DELA1(3),DELA2(3),DELF(7)
320 CHARACTER CV(2,40)
340 CHARACTER*1 P1(40),P2(40)
360 DATA P1/"A","@"/
380 DATA P2/"A","A","*","@"/
400 LIBRARY "SENSUB"
420 P1(2)="@"
425 ICNT=0
440*
460*
480*
500       M(2)=.0656/.075
520       M(3)=.303/.075
540       M(4)=1./.075
560 PRINT,"M1,M2,M3,M4",M
580 A=1170.
1980  IDELA=3
2000 DELA1(3)=1.
2020 DELA1(2)=.303
2040 DELA1(1)=.0656
2060*
2080 DELA2(3)=1.
2100 DELA2(2)=175.
2120 DELA2(1)=15600.
2270*
2280*  LOOP FOR THE THREE M VALUES
2290*
2300 DO 400 II=2,4
2310 DO 300 I=1,10
2320 300 GN(1,I)=0.0
2330 IGN(1)=II-1
2340 GN(1,II-1)=M(II)*A
2580 IGD(1)=IDELA
2600 DO 140 J=1,3
2620 140GD(1,J)=DELA1(J)
2625 IGD(2)=IDELA
2627 DO 150 I=1,3
2628 150 GD(2,I)=DELA2(I)
2760 IGNP=1
```

SENSEM (continued)

```
2780 IGDP=2
2860 DO 180 I=1,40
2880 CV(1,I)=P1(I)
2900 180 CV(2,I)=P2(I)
2920 PRINT,"P1",(P1(I),I=1,10)
2960 PRINT,"P2",(P2(I),I=1,10)
3060 WN=6
3080 WI=.001
3100*
3120 CALL SENSUB
3140 400 CONTINUE
3160*
3180 END
```

SENSEAO

```
100***  MAIN PROGRAM TO COMPUTE THE FREQUENCY
120***    RESPONSE OF E(S)
130***   WITH RESPECT TO OBSERVER GAIN AND THETA
140***
160***
180         COMMON GN,GD,IGN,IGD,WN,WI,IGNP,IGDP,CV
200         IMPLICIT DOUBLE PRECISION(A-H,O-Z)
220         DOUBLE PRECISION K1,K2,K3,K4,M
240         INTEGER IGN(20),IGD(20),WN
260         DIMENSION GN(20,10),GD(20,10)
265         DIMENSION F(3,4),H(3,7)
266         DIMENSION M(3),DELA1(3),DELA2(3),DELF(7)
280 CHARACTER CV(2,40)
281 CHARACTER*1 P1(40),P2(40)
285 DATA P2/"A","@"/
287 DATA P1/2*"A","+","A","*","@"/
300 LIBRARY "SENSUB"
340*
360*
380*
400         M(1)=.0656/.075
420         M(2)=.303/.075
440         M(3)=1./.075
460 PRINT,"M2,M3,M4",M
480 A=1170.
500 B=.0656*15600.
520 C=.303*15600. + .0656*175.
540 D=15600.+0.0656+.303*175.
560 E=175.303
580 PRINT,"A,B,C,D",A,B,C,D
600 PRINT,"INPUT,N"
610 INPUT,N
620 DO 400 IK=1,N
640 PRINT,"R COEFFS"
660 INPUT,R2,R3,R4,R5
680 PRINT,"R2,R3,R4,R5",R2,R3,R4,R5
700 K4 = R2-E
720 K3 = (B-R5)/B
740 K2 = (C*(R5/B)-R4)/B
760 K1 = (D*(R5/B)-C*K2-R3)/B
780 PRINT,"K1,K2,K3,K4",K1,K2,K3,K4
800*
1090 IDELA=3
1091*
1093 DELA1(3)=1.
1094 DELA1(2)=.303
1095 DELA1(1)=.0656
1097*
1098 DELA2(3)=1.
1099 DELA2(2)=175.
```

SENSEAO (continued)

```
1100 DELA2(1)=15600.
1102*
1104 IDELF=5
1105 DELF(5)=1
1106 DELF(4)=R2
1107 DELF(3)=R3
1108 DELF(2)=R4
1109 DELF(1)=R5
1110*
1130 IGN(1)=2
1140 GN(1,1)=M(1)*K3
1150 GN(1,2)=M(1)*K2+M(2)*K3
1390*
1400 IGN(3)=1
1410 GN(3,1)=-1170.
1420*
1430*
1440*
1456 IGN(2)=IDELA
1480 DO 140 J=1,3
1485 GN(2,J)=-M(3)*DELA1(J)
1490 140 CONTINUE
1520*
1530 IGD(1)=IDELF
1540 DO 160 I=1,5
1550 160 GD(1,I)=DELF(I)
1560*
1570*
1580 IGNP=3
1590 IGDP=1
1615*
1620 DO 180 I=1,40
1630 CV(1,I)=P1(I)
1640 180 CV(2,I)=P2(I)
1645 PRINT,"P1",(P1(I),I=1,10)
1646 PRINT,"P1",(P1(I),I=11,20)
1647 PRINT,"P2",(P2(I),I=1,10)
1660 WN=7
1661 WI=.01
1670*
1680 CALL SENSUB
1685 400 CONTINUE
1690*
1700 END
```

SENSEAS

```
100***  MAIN PROGRAM TO COMPUTE THE FREQUENCY
120***    RESPONSE OF E(S)
130***   WITH RESPECT TO SENSOR GAIN AND THETA DOT
140***
160***
180        COMMON GN,GD,IGN,IGD,WN,WI,IGNP,IGDP,CV
200        IMPLICIT DOUBLE PRECISION(A-H,O-Z)
220        DOUBLE PRECISION K1,K2,K3,K4,M
240        INTEGER IGN(20),IGD(20),WN
260        DIMENSION GN(20,10),GD(20,10)
265        DIMENSION F(3,4),H(3,7)
266        DIMENSION M(3),DELA1(3),DELA2(3),DELF(7)
280 CHARACTER CV(2,40)
281 CHARACTER*1 P1(40),P2(40)
285 DATA P2/2*"A","*","A","*","@"/
287 DATA P1/2*"A","*",2*"A","*","+","A","+","A","*","@"/
300 LIBRARY "SENSUB"
340*
360*
380*
400        M(1)=.0656/.075
420        M(2)=.303/.075
440        M(3)=1./.075
460 PRINT,"M2,M3,M4",M
480 A=1170.
500 B=.0656*15600.
520 C=.303*15600. + .0656*175.
540 D=15600.+0.0656+.303*175.
560 E=175.303
580 PRINT,"A,B,C,D",A,B,C,D,E
600 PRINT,"INPUT,N"
610 INPUT,N
620 DO 400 IK=1,N
640 PRINT,"R COEFFS"
660 INPUT,R2,R3,R4,R5
680 PRINT,"R2,R3,R4,R5",R2,R3,R4,R5
700 K4 = R2-E
720 K3 = (B-R5)/B
740 K2 = (C*(R5/B)-R4)/B
760 K1 = (D*(R5/B)-C*K2-R3)/B
780 PRINT,"K1,K2,K3,K4",K1,K2,K3,K4
800*
1090 IDELA=3
1091*
1093 DELA1(3)=1.
1094 DELA1(2)=.303
1095 DELA1(1)=.0656
1097*
1098 DELA2(3)=1.
1099 DELA2(2)=175.
```

SENSEAS (continued)

```
1100 DELA2(1)=15600.
1102*
1104 IDELF=5
1105 DELF(5)=1
1106 DELF(4)=R2
1107 DELF(3)=R3
1108 DELF(2)=R4
1109 DELF(1)=R5
1110*
1130 IGN(1)=2
1140 GN(1,1)=M(1)*K3
1150 GN(1,2)=M(1)*K2+M(2)*K3
1390*
1400 IGN(6)=1
1410 GN(6,1)=1170.
1420*
1430*
1440*
1450 IGD(1)=1
1455 IGN(3)=IDELA
1456 IGN(2)=IDELA
1457 IGN(4)=IDELA
1460 IGD(2)=IDELA
1480 DO 140 J=1,3
1485 GN(3,J)=-M(3)*DELA1(J)
1490 140 CONTINUE
1495 GD(1,1)=1.
1500 DO 150 J=1,3
1505 GN(2,J)=DELA2(J)
1507 GN(4,J)=DELA2(J)
1510 150 GD(2,J)=DELA2(J)
1520*
1530 IGD(3)=IDELF
1535 IGN(5)=IDELF
1540 DO 160 I=1,5
1545 GN(5,I)=M(3)*DELF(I)
1550 160 GD(3,I)=DELF(I)
1560*
1570*
1580 IGNP=6
1590 IGDP=3
1615*
1620 DO 180 I=1,40
1630 CV(1,I)=P1(I)
1640 180 CV(2,I)=P2(I)
1645 PRINT,"P1",(P1(I),I=1,10)
1646 PRINT,"P1",(P1(I),I=11,20)
1647 PRINT,"P2",(P2(I),I=1,10)
1660 WN=7
1661 WI=.01
```

SENSEAS (continued)

```
1670*
1680 CALL SENSUB
1685 400 CONTINUE
1690*
1700 END
```

SENSEZ1

```
100***  MAIN PROGRAM TO COMPUTE THE FREQUENCY
120***     RESPONSE W/R TO ZETA 1
140***
160***
180        COMMON GN,GD,IGN,IGD,WN,WI,IGNP,IGDP,CV
181        COMMON ICNT
200        IMPLICIT DOUBLE PRECISION(A-H,O-Z)
220        DOUBLE PRECISION K1,K2,K3,K4,M
240        INTEGER IGN(20),IGD(20),WN
260        DIMENSION GN(20,10),GD(20,10)
280        DIMENSION CC(4,7)
300        DIMENSION M(4),DELA1(3),DELA2(3),DELF(7)
320 CHARACTER CV(2,40)
340 CHARACTER*1 P1(40),P2(40)
360 DATA P2/3*"A",2*"*",2*"A","*","*","A","*","@"/
380 DATA P1/3*"A",2*"+","A","*",2*"A",2*"*","@"/
400 LIBRARY "SENSUB"
402 ICNT=0
420 P2(4)="*"
440 P2(5)="*"
460*
480*
500*
520        M(2)=.0656/.075
540        M(3)=.303/.075
560        M(4)=1./.075
580 PRINT,"M1,M2,M3,M4",M
600 A=1170.
620 B=.0656*15600.
640 C=.303*15600. + .0656*175.
660 D=15600.+0.0656+.303*175.
680 E=175.303
700 PRINT,"A,B,C,D",A,B,C,D,E
705 PRINT,"INPUT,N"
710 INPUT,N
720 DO 400 I=1,N
722 PRINT,"R COEFFS"
740 INPUT,R2,R3,R4,R5
800 PRINT,"R2,R3,R4,R5",R2,R3,R4,R5
820 K4 = R2-E
840 K3 = (B-R5)/B
860 K2 = (C*(R5/B)-R4)/B
880 K1 = (D*(R5/B)-C*K2-R3)/B
900 PRINT,"K1,K2,K3,K4",K1,K2,K3,K4
920*
940   IH2X=7
960 CC(2,7)=K2
980 CC(2,6) = K3 + K2*E
1000 CC(2,5) = K4 + K3*E + K2*D
1020 CC(2,4) = -K1*B
```

SENSEZ1 (continued)

```
1040*
1060 CC(3,7) = K3
1080 CC(3,6) = K3*E+K4
1100 CC(3,5) = -K1*B - K2*C
1120 CC(3,4) = -K2*B
1140*
1160 CC(4,7) = K4
1180 CC(4,6) = -(K1*B + K2*C + K3*D)
1200 CC(4,5) = -K2*B - K3*C
1220 CC(4,4) = -K3*B
1240*
1260*
1280  IDELA=3
1300*
1320 DELA1(3)=1.
1340 DELA1(2)=.303
1360 DELA1(1)=.0656
1380*
1400 DELA2(3)=1.
1420 DELA2(2)=175.
1440 DELA2(1)=15600.
1460*
1480 IDELF=5
1500 DELF(5)=1
1520 DELF(4)=R2
1540 DELF(3)=R3
1560 DELF(2)=R4
1580 DELF(1)=R5
1600*
1620*
1640 IGN(1)=7
1660 IGN(2)=7
1680 IGN(3)=7
1700 DO 100 I=2,4
1720 DO 100 J=4,7
1740 100 GN(I-1,J)=M(I)*CC(I,J)
1760*
1780 IGN(4)=1
1800 GN(4,1)=.5915*A
1820*
1840 IGN(5)=2
1860 GN(5,2)=2.*SQRT(.0656)
1880*
1900 IGN(6)=3
1920 DO 110 I=1,3
1940 110 GN(6,I)=DELA2(I)
1960*
1980*
2000*
2020*
```

SENSEZ1 (continued)

```
2040 IGD(1)=IDELA
2060 IGD(2)=IDELA
2080 IGD(3)=IDELA
2100 IGD(4)=IDELA
2120 DO 140 J=1,3
2140 GD(2,J)=DELA1(J)
2160 140GD(1,J)=DELA1(J)
2180 DO 150 J=1,3
2200 GD(3,J)=DELA2(J)
2220 150 GD(4,J)=DELA2(J)
2240*
2260 IGD(5)=IDELF
2280 DO 160 I=1,5
2300 160 GD(5,I)=DELF(I)
2320*
2340 IGD(6)=2
2360 GD(6,2)=1
2380*
2400 IGNP=6
2420 IGDP=6
2440*
2460 DO 180 I=1,40
2480 CV(1,I)=P1(I)
2500 180 CV(2,I)=P2(I)
2520 PRINT,"P1",(P1(I),I=1,17)
2540 PRINT,"P2",(P2(I),I=1,17)
2560 WN=6
2580 WI=.001
2600*
2640 CALL SENSUB
2645 400 CONTINUE
2660 END
```

SENSEW1

```
*
IGN(1)=7
IGN(2)=7
IGN(3)=7
DO 100 I=2,4
DO 100 J=4,7
100 GN(I-1,J)=M(I)*CC(I,J)
*
IGN(4)=1
GN(4,1)=A*SQRT(.0656)
*
IGN(5)=2
GN(5,2)=2.*(.5915)
GN(5,1)=2.*SQRT(.0656)
*
IGN(6)=3
DO 110 I=1,3
110 GN(6,I)=DELA2(I)
*
*
*
*
IGD(1)=IDELA
IGD(2)=IDELA
IGD(3)=IDELA
IGD(4)=IDELA
DO 140 J=1,3
GD(2,J)=DELA1(J)
140GD(1,J)=DELA1(J)
DO 150 J=1,3
GD(3,J)=DELA2(J)
150 GD(4,J)=DELA2(J)
*
IGD(5)=IDELF
DO 160 I=1,5
160 GD(5,I)=DELF(I)
*
IGD(6)=2
GD(6,2)=1
*
IGNP=6
IGDP=6
*
```

SENSEW2

```
*
IGN(1)=7
IGN(2)=7
IGN(3)=7
DO 100 I=2,4
DO 100 J=4,7
100 GN(I-1,J)=M(I)*CC(I,J)
*
IGN(4)=1
GN(4,1)=A*SQRT(15600.)
*
IGN(5)=2
GN(5,2)=2.*.70056
GN(5,1)=2.*124.9
*
IGN(6)=3
DO 110 I=1,3
110 GN(6,I)=DELA1(I)
*
*
*
*
IGD(1)=IDELA
IGD(2)=IDELA
IGD(3)=IDELA
IGD(4)=IDELA
DO 140 J=1,3
GD(2,J)=DELA1(J)
140GD(1,J)=DELA1(J)
DO 150 J=1,3
GD(3,J)=DELA2(J)
150 GD(4,J)=DELA2(J)
*
IGD(5)=IDELF
DO 160 I=1,5
160 GD(5,I)=DELF(I)
*
IGD(6)=2
GD(6,2)=1
*
IGNP=6
IGDP=6
*
```

SENSEZ2

```
*
*
IGN(1)=7
IGN(2)=7
IGN(3)=7
DO 100 I=2,4
DO 100 J=4,7
100 GN(I-1,J)=M(I)*CC(I,J)
*
IGN(4)=1
GN(4,1)=A*.700561
*
IGN(5)=2
GN(5,2)=2.*124.9
*
IGN(6)=3
DO 110 I=1,3
110 GN(6,I)=DELA1(I)
*
*
*
*
IGD(1)=IDELA
IGD(2)=IDELA
IGD(3)=IDELA
IGD(4)=IDELA
DO 140 J=1,3
GD(2,J)=DELA1(J)
140GD(1,J)=DELA1(J)
DO 150 J=1,3
GD(3,J)=DELA2(J)
150 GD(4,J)=DELA2(J)
*
IGD(5)=IDELF
DO 160 I=1,5
160 GD(5,I)=DELF(I)
*
IGD(6)=2
GD(6,2)=1
*
IGNP=6
IGDP=6
*
```

SENSEK

```
***
**  HERE FOR THE VARIOUS K VALUES, II=1,4
*
II=1
* DO 400 II=1,4
IGN(1)=IF(2,II)
DO 70 I=1,IGN(1)
70 GN(1,I)=M(2)*F(2,II,I)
IGN(2)=IF(3,II)
DO 80 I=1,IGN(2)
80 GN(2,I)=M(3)*F(3,II,I)
IGN(3)=IF(4,II)
DO 90 I=1,IGN(3)
90 GN(3,I)=M(4)*F(4,II,I)
IGN(4)=4
GN(4,4)=K(II)*A
*
IGD(1)=IDELA
DO 140 J=1,3
140GD(1,J)=DELA1(J)
IGD(4)=IDELA
DO 150 I=1,3
150 GD(4,I)=DELA2(I)
*
IGD(2)=IDELF
DO 160 I=1,5
160 GD(2,I)=DELF(I)
*
*
IGNP=4
IGDP=4
GD(3,2)=1
IGD(3)=2
*
```

SENSEZ10

```
100***  MAIN PROGRAM TO COMPUTE THE SENSITIVITY
120***     OF ECAP W/R TO ZETA1 OF THE OBSERVER
140***
160***
180        COMMON GN,GD,IGN,IGD,WN,WI,IGNP,IGDP,CV
185        COMMON ICNT
200        IMPLICIT DOUBLE PRECISION(A-H,O-Z)
220        DOUBLE PRECISION K(4),M
240        INTEGER IGN(20),IGD(20),WN
260        DIMENSION GN(20,10),GD(20,10)
280        DIMENSION F(4,4,4),IF(4,4)
300        DIMENSION M(4),DELA1(3),DELA2(3),DELF(7)
320 CHARACTER CV(2,40)
340 CHARACTER*1 P1(40),P2(40)
360 DATA P1/3*"A",2*"+",2*"A",2*"*","@"/
380 DATA P2/1HA,1HA,1HA,1H*,1H*,1HA,1H*,1H@/
400 LIBRARY "SENSUB"
425 ICNT=0
440*
460*
480*
500        M(2)=.0656/.075
520        M(3)=.303/.075
540        M(4)=1./.075
560 PRINT,"M1,M2,M3,M4",M
580 A=1170.
600 B=.0656*15600.
620 C=.303*15600. + .0656*175.
640 D=15600.+0.0656+.303*175.
660 E=175.303
680 PRINT,"A,B,C,D",A,B,C,D,E
700 PRINT,"INPUT,N"
710 INPUT,N
720 DO 400 JJ=1,N
730 PRINT,"R COEFFS"
740 INPUT,R2,R3,R4,R5
780 PRINT,"R2,R3,R4,R5",R2,R3,R4,R5
800 K(4) = R2-E
820 K(3) = (B-R5)/B
840 K(2) = (C*(R5/B)-R4)/B
860 K(1) = (D*(R5/B)-C*K(2)-R3)/B
880 PRINT,"K1,K2,K3,K4",K
900 DO 10 I=1,4
920 DO 10 J=1,4
940 DO 10 KL=1,4
960 10 F(I,J,KL)=0.
980**
1000 IF(2,1)=2
1020 F(2,1,1)=B*(K(3)-1)
1040 F(2,1,2)=B*K(2)
```

SENSEZ10 (continued)

```
1060*
1080 IF(2,2)=4
1100 F(2,2,2)=-B*K(1)-D*(K(3)-1)
1120 F(2,2,3)=E+K(4)
1140 F(2,2,4)=1
1160*
1180 IF(2,3)=3
1200 F(2,3,1)=-K(1)*B
1220 F(2,3,2)=K(2)*D + E+K(4)
1240 F(2,3,3)=1
1260*
1280 IF(2,4)=3
1300 F(2,4,2)=-K(3)+1
1320 F(2,4,3)=-K(2)
1340*
1360 IF(3,1)=2
1380 F(3,1,2)=B*(K(3)-1)
1400*
1420 IF(3,2)=2
1440 F(3,2,1)=B*(K(3)-1)
1460 F(3,2,2)=C*(K(3)-1)
1480*
1500 IF(3,3)=4
1520 F(3,3,1)=-B*K(2)
1540 F(3,3,2)=B*K(1)-C*K(2)
1560 F(3,3,3)=K(4)+E
1580 F(3,3,4)=1
1600*
1620 IF(3,4)=3
1640 F(3,4,3)=1-K(3)
1660*
1680 IF(4,1)=3
1700 F(4,1,3)=-B
1720*
1740 IF(4,2)=3
1760 F(4,2,2)=-B
1780 F(4,2,3)=-C
1800*
1820 IF(4,3)=3
1840 F(4,3,1)=-B
1860 F(4,3,2)=-C
1880 F(4,3,3)=-D
1900*
1920 IF(4,4)=4
1940 F(4,4,4)=1
1960*
1980  IDELA=3
2000 DELA1(3)=1.
2020 DELA1(2)=.303
2040 DELA1(1)=.0656
```

SENSEZ10 (continued)

```
2060*
2080 DELA2(3)=1.
2100 DELA2(2)=175.
2120 DELA2(1)=15600.
2140*
2160 IDELF=5
2180 DELF(5)=1
2200 DELF(4)=R2
2220 DELF(3)=R3
2240 DELF(2)=R4
2260 DELF(1)=R5
2279***
2280***
2283*
2300 II=4
2340 IGN(1)=IF(2,II)
2360 DO 70 I=1,IGN(1)
2380 70 GN(1,I)=M(2)*F(2,II,I)
2400 IGN(2)=IF(3,II)
2420 DO 80 I=1,IGN(2)
2440 80 GN(2,I)=M(3)*F(3,II,I)
2460 IGN(3)=IF(4,II)
2480 DO 90 I=1,IGN(3)
2500 90 GN(3,I)=M(4)*F(4,II,I)
2520 IGN(4)=2
2540 GN(4,2)=2*SQRT(.0653)*A*.5915
2560*
2580 IGD(1)=IDELA
2600 DO 140 J=1,3
2620 140GD(1,J)=DELA1(J)
2625 IGD(4)=IDELA
2626 IGN(5)=IDELA
2627 DO 150 I=1,3
2628 GD(4,I)=DELA2(I)
2629 150 GN(5,I)=DELA2(I)
2640*
2660 IGD(2)=IDELF
2680 DO 160 I=1,5
2700 160 GD(2,I)=DELF(I)
2720*
2740*
2760 IGNP=5
2780 IGDP=4
2800 GD(3,2)=1
2820 IGD(3)=2
2840*
2860 DO 180 I=1,40
2880 CV(1,I)=P1(I)
2900 180 CV(2,I)=P2(I)
2920 PRINT,"P1",(P1(I),I=1,10)
```

SENSEZ10 (continued)

```
2960 PRINT,"P2",(P2(I),I=1,10)
3060 WN=6
3080 WI=.001
3100*
3120 CALL SENSUB
3140 400 CONTINUE
3160*
3180 END
```

SENSEW10

```
2280***
2283*
2300 II=4
2340 IGN(1)=IF(2,II)
2360 DO 70 I=1,IGN(1)
2380 70 GN(1,I)=M(2)*F(2,II,I)
2400 IGN(2)=IF(3,II)
2420 DO 80 I=1,IGN(2)
2440 80 GN(2,I)=M(3)*F(3,II,I)
2460 IGN(3)=IF(4,II)
2480 DO 90 I=1,IGN(3)
2500 90 GN(3,I)=M(4)*F(4,II,I)
2520 IGN(4)=2
2540 GN(4,2)=A*.303
2541 GN(4,1)=2*A*.0656
2560*
2580 IGD(1)=IDELA
2600 DO 140 J=1,3
2620 140GD(1,J)=DELA1(J)
2625 IGD(4)=IDELA
2626 IGN(5)=IDELA
2627 DO 150 I=1,3
2628 GD(4,I)=DELA2(I)
2629 150 GN(5,I)=DELA2(I)
2640*
2660 IGD(2)=IDELF
2680 DO 160 I=1,5
2700 160 GD(2,I)=DELF(I)
2720*
2740*
2760 IGNP=5
2780 IGDP=4
2800 GD(3,2)=1
2820 IGD(3)=2
2840*
```

SENSEW20

```
2280***
2283*
2300 II=4
2340 IGN(1)=IF(2,II)
2360 DO 70 I=1,IGN(1)
2380 70 GN(1,I)=M(2)*F(2,II,I)
2400 IGN(2)=IF(3,II)
2420 DO 80 I=1,IGN(2)
2440 80 GN(2,I)=M(3)*F(3,II,I)
2460 IGN(3)=IF(4,II)
2480 DO 90 I=1,IGN(3)
2500 90 GN(3,I)=M(4)*F(4,II,I)
2520 IGN(4)=2
2540 GN(4,2)=A*175
2541 GN(4,1)=2*A*15600
2560*
2580 IGD(1)=IDELA
2600 DO 140 J=1,3
2610 GN(5,J)=DELA1(J)
2620 140GD(1,J)=DELA1(J)
2625 IGD(4)=IDELA
2626 IGN(5)=IDELA
2627 DO 150 I=1,3
2628 GD(4,I)=DELA2(I)
2629 150 CONTINUE
2640*
2660 IGD(2)=IDELF
2680 DO 160 I=1,5
2700 160 GD(2,I)=DELF(I)
2720*
2740*
2760 IGNP=5
2780 IGDP=4
2800 GD(3,2)=1
2820 IGD(3)=2
2840*
```

```
2280***
2283*
2300 II=4
2340 IGN(1)=IF(2,II)
2360 DO 70 I=1,IGN(1)
2380 70 GN(1,I)=M(2)*F(2,II,I)
2400 IGN(2)=IF(3,II)
2420 DO 80 I=1,IGN(2)
2440 80 GN(2,I)=M(3)*F(3,II,I)
2460 IGN(3)=IF(4,II)
2480 DO 90 I=1,IGN(3)
2500 90 GN(3,I)=M(4)*F(4,II,I)
2520 IGN(4)=2
2540 GN(4,2)=2*A*.700561*SQRT(15600.)
2560*
2580 IGD(1)=IDELA
2600 DO 140 J=1,3
2610 GN(5,J)=DELA1(J)
2620 140GD(1,J)=DELA1(J)
2625 IGD(4)=IDELA
2626 IGN(5)=IDELA
2627 DO 150 I=1,3
2628 GD(4,I)=DELA2(I)
2629 150 CONTINUE
2640*
2660 IGD(2)=IDELF
2680 DO 160 I=1,5
2700 160 GD(2,I)=DELF(I)
2720*
2740*
2760 IGNP=5
2780 IGDP=4
2800 GD(3,2)=1
2820 IGD(3)=2
2840*
```

SENSUB

```
100        SUBROUTINE SENSUB
120        COMMON GNUM,GDEM,IGN,IGD,WN,WI
140        COMMON IGNP,IGDP,C
160        COMMON ICNT,IOUT1,IOUT2
180        IMPLICIT DOUBLE PRECISION(A-H,O-Z)
200        COMPLEX P(20),S,XM1,XM2,R1,R2,R3
220        INTEGER OUT,WN,IGN(20),IGD(20)
240        REAL PA,PB,MAG(200,2),PHAS(200,2)
260        DIMENSION GNUM(20,10),GDEM(20,10),WX(20),WS(21)
280        DIMENSION A1(20),A2(20),B1(20),B2(20)
300        DATA WS/1,1.25,1.3,1.4,1.5,1.6,1.7,1.8,2,2.25,2.5,2.75,3,3.5,4,
320       & 4.5,5,6,7,8,9/
340        CHARACTER YES,FILE1,FILE2
360        CHARACTER FILE3,C(2,40)
380        NCNT=21
420 NSTEP=2
600*
620* OUTPUT TO FILE OR TTY
640*
650        OUT=0
660        IF(ICNT .NE. 0)GO TO 10
680        5 CONTINUE
700        PRINT,"OUTPUT TO A FILE?"
720        INPUT,YES
740        IF(YES .NE. "YES") GO TO 10
760        PRINT,"FILE NAME IS ?"
780        INPUT,FILE2
800       PRINT,"FILE NAME IS ?"
820       INPUT,FILE3
840       OPENFILE 4,FILE3
860       REWIND 4
880       ENDFILE 4
900       IOUT2=4
920        OPENFILE 3,FILE2
940        REWIND 3
960        ENDFILE 3
980       IOUT1=3
1000 10      CONTINUE
1020        ICNT=1
1040*
1045 ICNT=1
1060        WRITE(OUT,610)
1080        WRITE(OUT,620)
1100        WRITE(OUT,640)
1120        DO 45 J=1,IGNP
1140        WRITE(OUT,650)(GNUM(J,I),I-1,I=1,IGN(J))
1160 45     CONTINUE
1180        WRITE(OUT,660)
1200        DO 55 J=1,IGDP
1220        WRITE(OUT,650)(GDEM(J,I),I-1,I=1,IGD(J))
```

SENSUB (continued)

```
55    CONTINUE
      WRITE(OUT,850)
      WRITE(OUT,610)
      WRITE(OUT,800)
      K1=0
      NCNT2=0
100   DO 250 KZ=1,WN
      DO 200 K=1,NCNT,NSTEP
      W=WS(K)*WI
      W2=-W*W
      K1=K1+1
      CALL PQSD(A1,B1,IGNP,W2,GNUM,IGN)
      CALL PQSD(A2,B2,IGDP,W2,GDEM,IGD)
      DO 105 I=1,IGNP
105   A1(I) = A1(I)*W
      DO 107 I=1,IGDP
107   A2(I) = A2(I)*W
      PHAS(K1,1)=W
      MAG(K1,1)=W
      ISW=1
      DO 108 I=1,IGNP
108   P(I) = CMPLX(SNGL(B1(I)),SNGL(A1(I)))
110   S=CMPLX(0.,0.)
      J=0
      I=0
120    I=I+1
125   J=J+1
      IF(C(ISW,J) .EQ. "A") GO TO 130
      IF(C(ISW,J) .EQ. "*") GO TO 135
      IF(C(ISW,J) .EQ. "+") GO TO 140
      IF(C(ISW,J) .EQ. "-") GO TO 145
      IF(C(ISW,J) .EQ. "S") GO TO 150
      IF(C(ISW,J) .EQ. "R") GO TO 155
      IF(C(ISW,J) .EQ. "@") GO TO 160
      PRINT,"UNKNOWN CHARACTER",C(ISW,J)
      STOP
*
130   R3=R2
      R2=R1
      R1=P(I)
      GO TO 120
*
135   R1=R1*R2
137   R2=R3
      GO TO 125
*
140   R1=R1+R2
      GO TO 137
*
145   R1=R1-R2
```

SENSUB (continued)

```
      GO TO 137

150   S=R1
      GO TO 125

155   R3=R2
      R2=R1
      R1=S
      GO TO 125

160   IF(ISW .EQ. 2) GO TO 170
      ISW=2
      DO 167 I=1,IGDP
167   P(I) = CMPLX(SNGL(B2(I)),SNGL(A2(I)))
      XM1=R1
      GO TO 110

170    XM2=R1
      CALL PHASE(REAL(XM1),AIMAG(XM1),SNGL(W),PA)
      CALL PHASE(REAL(XM2),AIMAG(XM2),SNGL(W),PB)
      PHAS(K1,2) = (PA-PB)*180./3.141593
      XM1=CMPLX(REAL(XM1),AIMAG(XM1)*SNGL(W))
      XM2=CMPLX(REAL(XM2),AIMAG(XM2)*SNGL(W))
      MAG(K1,2) = CABS(XM1)/CABS(XM2)
      WRITE(OUT,810)W,MAG(K1,2),PHAS(K1,2)
      NCNT2=NCNT2+1
      IF(NCNT2 .LE. 44) GO TO 200
      NCNT2=0
      WRITE(OUT,670)
      WRITE(OUT,610)
      WRITE(OUT,800)
200    CONTINUE
      WRITE(OUT,820)
      WI=WI*10
250    CONTINUE
     IF(NCNT .EQ. 1) GO TO 270
      NCNT=1
      WN=1
      GO TO 100
270    CONTINUE
      NCNT2=(55-NCNT2)/3
      IF(IOUT1 .EQ. 0)GO TO 320
     DO 300 I=1,K1
300   WRITE(IOUT1,735)MAG(I,1),MAG(I,2)
      DO 310 I=1,K1
310   WRITE(IOUT2,735)PHAS(I,1),PHAS(I,2)
      WRITE(IOUT1,745)
      WRITE(IOUT2,745)
      WRITE(OUT,850)
320   CONTINUE
```

SENSUB (continued)

```
RETURN
*
*
*
*
600      FORMAT(/,30("="),"INPUT DATA",30("=")/)
690      FORMAT(V)
735      FORMAT(E13.5," , ",E12.5)
745      FORMAT("  1E37   1E37")
760      FORMAT(1H ,"NUMBER OF DECADES AND INITIAL FREQUENCY")
700      FORMAT(1H ,"GAIN OF G(S)=")
710      FORMAT(1H ,"HOW MANY POLYNOMINALS IN THE NUMERATOR OF G(S)")
720      FORMAT(1H ,"HOW MANY POLYNOMINALS IN THE DENOMINATOR OF G(S)")
610      FORMAT(1H ,70("=")///)
620      FORMAT(1H ,"G(S)")
640      FORMAT(1H ,"NUMERATOR TERMS")
660      FORMAT(1H ,"DENOMINATOR TERMS")
670       FORMAT(/)
650      FORMAT(1H ,4(E11.5,"*S**",I2,"+")/5(5X,3(E11.5,"*S**",I2,"+")/))
800      FORMAT(1H ,"FREQUENCY",3X,"MDB OP-LOOP",3X,"PHASE OP-LP",3X,
       &)
810      FORMAT(1X,G8.3,G14.6,G14.6)
820       FORMAT(1H )
850      FORMAT(////)
        END
*
*
*
        SUBROUTINE TEREAD(Y,J,IN,M,L1,C)
        DOUBLE PRECISION Y(20,10)
        DIMENSION J(20)
        CHARACTER C(2,40)
        READ(IN,999)M
        IF(M .EQ. 0)GO TO 70
         WRITE(OUT,998)
        DO 40 I=1,M
        READ(IN,999)K,(Y(I,K+1-L),L=1,K)
        J(I) = K
40       CONTINUE
        IF(M .EQ. 1)GO TO 60
        WRITE(OUT,997)
        READ(IN,999)(C(L1,I),I=1,40)
45      CONTINUE
999      FORMAT(V)
997     FORMAT(" ENTER THE CHARACTER STRING - ONE LINE")
998       FORMAT(1H ,"ENTER NUMBER OF COEFFS THEN COEFFS,HIGH",/,5X,
       & " TO LOW, INCLUDING ZEROS FOR EACH POLYNOMINAL")
50      RETURN
60      C(L1,1)="A"
        C(L1,2)="@"
```

SENSUB (continued)

```
        GO TO 50
 70     M=1
        Y(1,1)=1.
        GO TO 60
        END
*
*
*
        SUBROUTINE PHASE(R,I,W,P)
        REAL I
        PI=3.1415926
 10         IF(I)20,30,40
 20         IF(R)100,120,130
 30         IF(R)100,140,140
 40         IF(R)170,150,160
 *
 100      P=PI+ATAN(I/R)
 110      RETURN
 120      P=PI+PI/2.
        GO TO 110
 130      P=2*PI+ATAN(I/R)
        GO TO 110
 140      P=0.
        GO TO 110
 150      P=PI/2.
        GO TO 110
 160      P=ATAN(I/R)
        GO TO 110
 170      P=PI+ATAN(I/R)
        GO TO 110
        END
*
*
*
 SUBROUTINE PQSD(A,B,M,Q,X,ID)
        DOUBLE PRECISION X(20,10),A(20),B(20),Q
        DIMENSION ID(20)
        DO 3 K=1,M
 A(K)=0
 B(K)=0
 J=ID(K)
  1  IF(J) 3,3,2
  2    Z=B(K)
  B(K)=Q*A(K)+X(K,J)
  A(K)=Z
  J=J-1
  GO TO 1
  3  CONTINUE
       RETURN
  END
```

POLYPAK

```
*   POLYPAK
*
*
*       POLYPAK IS A PACKAGE OF *FORTRAN* SUBROUTINES FOR POLYNOMINAL
*    OPERATIONS.    E.E. MITCHELL 9/78
*
*       ALL COEFFICIENT VECTORS HAVE A DIMENSION OF 20.
*
*      EACH VECTOR OF COEFFICIENTS MUST BE ORDERED FROM *** LOW ORDER
*    TO HIGH ORDER*** AND WITH EACH VECTOR OF COEFFICIENTS THERE MUST
*    BE AN INTEGER TELLING THE *NUMBER OF COEFFICIENTS* IN THE POLY.
*
*
*
*       THE USAGE IS AS FOLLOWS:
*
*  CALL PADD(X,IX,Y,IY,Z,IZ)
*       POLY Z IS ADDED TO POLY Y AND GIVES POLY X
*
*  CALL PSUB(Z,IZ,X,IX,Y,IY)
*       POLY Y IS SUBTRACTED FROM POLY X - RESULTS IN POLY Z
*
*
*  CALL PMPY(Z,IZ,X,IX,Y,IY)
*        POLY X IS MULTIPLIED TIMES POLY Y TO GIVE Z
*
*   *   *   *   *   *   *   *   *   *   *   *   *   *   *   *
***  THIS PACKAGE IS A COLLECTION OF FORTRAN CALLABLE
***  SUBROUTINES FOR POLYNOMINAL OPERATIONS
*
*
         SUBROUTINE PADD(X,IX,Y,IY,Z,IZ)
* ADD TWO POLYS
         DIMENSION X(20),Y(20),Z(20)
         DO 10 I=1,19
  10      X(I)=0.
         DO 20 I=1,IY
  20      X(I)=Y(I)
         DO 30 I=1,IZ
  30      X(I)=X(I)+Z(I)
         IF(IY-IZ)40,40,60
  40      IX=IZ
  50      RETURN
  60      IX=IY
         GO TO 50
         END
*   *   *   *   *   *   *
*
          SUBROUTINE PSUB(Z,IZ,X,IX,Y,IY)
*
```

POLYPAK (continued)

```
* SUBTRACT ONE POLY FROM ANOTHER
DIMENSION X(20),Y(20),Z(20)
N=IX
IF(IX-IY) 10,20,20
10 N=IY
20 IF( N ) 90,90,30
30 DO 80 I =1,N
IF( I-IX ) 40,40,60
40 IF( I-IY ) 50,50,70
50 Z(I) = X(I) - Y(I)
GO TO 80
60 Z(I) = -Y(I)
GO TO 80
70 Z(I) = X(I)
80 CONTINUE
90 IZ = N
RETURN
END
*
*    *    *    *    *    *    *
SUBROUTINE PMPY(Z,IZ,X,IX,Y,IY)
*
* MULTIPLIES TWO POLYS
DIMENSION Z(20),X(20),Y(20)
IZ=IX+IY-1
DO 30 I=1,IZ
30 Z(I)=0.
DO 40 I=1,IX
DO 40 J=1,IY
K=I+J-1
40 Z(K)=X(I)*Y(J)+Z(K)
RETURN
END
*
*    *    *    *    *    *    *
*
```

APPENDIX C. COMPUTER PROGRAMS FOR THE TIME RESPONSE OF THE SENSITIVITY EQUATIONS

In Appendix B, programs were listed which generated the polynomials of the sensitivity equations for each of the parameters.

Several very small programs were then written, a typical one being STRAJP, which is listed below.

```
STRAJP                  09/24/78              19:20:58


100***    MAIN PROGRAM TO COMPUTE THE TIME EQUATIONS
110***       FOR THE SENSITIVITY OF E WITH RESPECT
120***       TO THE PARAMETERS Z1, Z2, W1, W2
200*
400 LIBRARY "TRAJEQ"
478 PRINT, "*****"
479 PRINT, "*****"
480 PRINT, "RUN FOR PARAMETER Z1**"
481 PRINT, "***"
2630 C(1,40)="Z1**"
2635 C(1,39)="DTRAJZ1"
2640 CALL TRAJEQ
```

These small programs were overlaid on the sensitivity programs of Appendix B. The combined program then called subroutine TRAJEQ which output the polynomial coefficients into a file. The program INTEGX would then read the coefficients from the file and run a time solution.

The subroutine TRAJEQ and program INTEGX are listed next.

TRAJEQ

```
100        SUBROUTINE TRAJEQ
120        COMMON GNUM,GDEM,IGN,IGD,WN,WI
140        COMMON IGNP,IGDP,C
160        COMMON ICNT,IOUT1,IOUT2
180*       IMPLICIT DOUBLE PRECISION(A-H,O-Z)
200        INTEGER IG(20),IR3,IR2,IR1,IS
220        INTEGER OUT,WN,IGN(20),IGD(20)
240        REAL PA,PB,MAG(200,2),PHAS(200,2)
260        DIMENSION GNUM(20,10),GDEM(20,10),WX(20),WS(21)
280        DIMENSION S(20), A1(20),A2(20),B1(20),B2(20)
300        DIMENSION GX(20,10),R1(20),R2(20),R3(20)
320        LIBRARY "L.ES***:POLYPAK"
340        CHARACTER YES,FILE1,FILE2
360        CHARACTER FILE3,C(2,40)
380*
400* OUTPUT TO FILE OR TTY
420*
440        OUT=0
460        IF(ICNT .NE. 0)GO TO 10
480*       OUT=0
500*       5 CONTINUE
520*       PRINT,"OUTPUT TO A FILE?"
540*       INPUT,YES
560*       IF(YES .NE. "YES") GO TO 10
580*       PRINT,"FILE NAME IS ?"
600*       INPUT,FILE2
620       OPENFILE 4,C(1,39),"NUMERIC"
640        IOUT1 = 4
660        REWIND 4
680        ENDFILE 4
700 10     CONTINUE
720*
740 ICNT=1
760        WRITE(OUT,610)
780        WRITE(OUT,620)
800        WRITE(OUT,640)
820        DO 45 J=1,IGNP
840*       WRITE(OUT,650)(GNUM(J,I),I-1,I=1,IGN(J))
860 45     CONTINUE
880        WRITE(OUT,660)
900        DO 55 J=1,IGDP
920*       WRITE(OUT,650)(GDEM(J,I),I-1,I=1,IGD(J))
940 55     CONTINUE
960        WRITE(OUT,850)
980        WRITE(OUT,610)
1000 ISW=1
1020 IR1=1
1040 IR2=1
1060 IR3=1
1080 R1(1)=1
```

TRAJEQ (continued)

```
1100 R2(1)=1
1120 R3(1)=1
1140 DO 108 I=1,IGNP
1160 IG(I) = IGN(I)
1180 DO 108 J=1,IG(I)
1200 108 GX(I,J) = GNUM(I,J)
1220 110 CONTINUE
1240       J=0
1260       I=0
1280 120    I=I+1
1300 125   J=J+1
1320       IF(C(ISW,J) .EQ. "A") GO TO 130
1340       IF(C(ISW,J) .EQ. "*") GO TO 135
1360       IF(C(ISW,J) .EQ. "+") GO TO 140
1380       IF(C(ISW,J) .EQ. "-") GO TO 145
1400       IF(C(ISW,J) .EQ. "S") GO TO 150
1420       IF(C(ISW,J) .EQ. "R") GO TO 155
1440       IF(C(ISW,J) .EQ. "@") GO TO 160
1460       PRINT,"UNKNOWN CHARACTER",C(ISW,J)
1480       STOP
1500*
1520 130 IR3=IR2
1540 DO 400 K=1,IR3
1560 400 R3(K) =R2(K)
1580 IR2= IR1
1600 DO 405 K=1,IR2
1620 405 R2(K) = R1(K)
1640 IR1= IG(I)
1660 DO 410 K=1,IR1
1680 410 R1(K) =GX(I,K)
1700 GO TO 120
1720*
1725*
1740 135 CALL PMPY(S,IS,R1,IR1,R2,IR2)
1760 DO 136 K=1,IS
1780 136 R1(K)=S(K)
1800 IR1=IS
1820 137 IR2 = IR3
1840 DO 420 K=1,IR2
1860 420 R2(K) = R3(K)
1880 GO TO 125
1900*
1920*
1940*
1960 140 CALL PADD(R1,IR1,R1,IR1,R2,IR2)
1980 GOTO 137
2000*
2020*
2040 145 CALL PSUB(R1,IR1,R1,IR1,R2,IR2)
2060 GO TO 137
```

TRAJEQ (continued)

```
2080*
2085*
2100 150 IS=IR1
2120 DO 430 K=1,IS
2140 430 S(K) = R1(K)
2160 GO TO 125
2180*
2200*
2220 155 IR3=IR2
2240 DO 440 K=1,IR3
2260 440 R3(K)=R2(K)
2280 IR2 = IR1
2300 DO 450 K=1,IR2
2320 450 R2(K) = R1(K)
2340 IR1 = IS
2360 DO 460 K=1,IR1
2380 460 R1(K) = S(K)
2400 GO TO 125
2420*
2440*
2460*
2480 160 IF(ISW .EQ. 2) GO TO 170
2500*
2520 DO 167 I=1,IGDP
2540 IG(I)=IGD(I)
2560 DO 167 J=1,IG(I)
2580 167 GX(I,J) = GDEM(I,J)
2600 WRITE(OUT,640)
2620 WRITE(OUT,645)
2640 168 WRITE(OUT,650)(R1(I),I-1,I=1,IR1)
2649 PPP=IR1+.00001
2650  WRITE(IOUT1) PPP,(R1(I),I=1,IR1)
2660 ISW=ISW+1
2680 GO TO (110,110,172),ISW
2700 170 WRITE(OUT,660)
2720 WRITE(OUT,645)
2740 GO TO 168
2760 172 WRITE(OUT,610)
2761 RETURN
2780 *
2800 *
2820 *
2840 600    FORMAT(/,30("="),"INPUT DATA",30("=")/)
2860 690    FORMAT(V)
2880 735    FORMAT(E13.5," , ",E12.5)
2900 745    FORMAT("1.E38 , 1.E38")
2920 610    FORMAT(1H ,70("=")///)
2940 620    FORMAT(1H ,"G(S)")
2960 640    FORMAT(1H ,"NUMERATOR TERMS")
2980 645 FORMAT(" COMBINED")
```

TRAJEQ (continued)

```
660     FORMAT(1H ,"DENOMINATOR TERMS")
670      FORMAT(/)
650     FORMAT(1H ,4(E13.7,"*S**",I2,"+")/5(5X,3(E13.7,"*S**",I2
820      FORMAT(1H )
850     FORMAT(////)
       END
```

INTEGX

```
1' INTEG
2'
3' DESCRIPTION--Solves a set of N first-order differential equations
4' of the form:
10'
20'       dX(I) = G(I) for I = 1,2,.......,N,
30'
40' when the initial conditons are known, by the fourth-order Runge Kutta
50' technique and provides a plot of T vs X(N1), X(N2), X(N3), and
60' X(N4) in that order depending on selection in DATA statement.
70'
80' INSTRUCTIONS--The functions, G(I), are to be inserted after line 1290
90' and before line 1320 in the following form:
100'
110'        1295   G(1) = WHATEVER IT IS
120'        1300   G(2) = WHATEVER IT IS
130'
140' The entire mathematical model should go in this section including
150' algebraic constraints.
160'
170'        1330 DATH,T2,T1,M
190'        1350 DATA H1,N1,N2,N3,N4
200'
220' WHERE H = Increment size
230'        T2 = Initial value of T
240'        T1 = Final value of T
250'        M = Number of variables to be plotted
260'        H1 = No. of increments of H between print outs (must be an
270'             integer.)
280'        N1,N2,N3,N4 identify the dependent variables to be printed out.
290'
320'
330'        T = Independent variable
340'        G(I) = Known functions = dX(I)/dT
350'
360'     *     *     *     *     MAIN PROGRAM     *     *     *     *
361 DIM Q(20),D(20)
362 PRINT"INPUT DATA FILE NAME"
363 INPUT F$
364 FILE#1: F$
365 READ#1: I7
366 FOR I=1 TO I7
367 READ#1: Q(I)
368 PRINT Q(I)
369 NEXT I
370 READ#1: I8
371 FOR I=1 TO I8
372 READ#1: D(I)
373 PRINT D(I)
374 NEXT I
```

INTEGX (continued)

```
375 I8=I8-1
380 DIM X(50),G(50),L(3,50),S(1000,6)
390 PRINT
400 FOR I=1 TO 72
410 PRINT "*";
420 NEXT I
430 PRINT
435 PRINT"PLOT FILE NAME"
436 INPUT T$
437 FILE#2: T$
438 SCRATCH#2
440 PRINT
450 GO TO 990
460 IF M=0 THEN 530
470 PRINT "PLOT SYMBOL";
480 FOR I=1 TO M
490 PRINT TAB(15*I+3);E$(I);
500 NEXT I
510 PRINT
520 PRINT
530 PRINT "T";
540 FOR I=1 TO 4
550 IF N(I)=0 THEN 600
560 PRINT TAB(15*I);"X(";N(I);")";
570 Z(I)=N(I)
580 N2=N2+1
590 NEXT I
600 PRINT
610 IF T<T2 THEN 970
620 GO TO 890
630 GO SUB 1290
640 FOR I=1 TO N
650 L(1,I)=H*G(I)
660 X(I)=X(I)+L(1,I)/2
670 NEXT I
680 T=T+H/2
690 GO SUB 1290
700 FOR I=1 TO N
710 L(2,I)=H*G(I)
720 X(I)=X(I)-L(1,I)/2+L(2,I)/2
730 NEXT I
740 GO SUB 1290
750 FOR I=1 TO N
760 L(3,I)=H*G(I)
770 X(I)=X(I)-L(2,I)/2+L(3,I)
780 NEXT I
790 T=T+H/2
800 GO SUB 1290
810 FOR I=1 TO N
820 X(I)=X(I)-L(3,I)+(L(1,I)+2*L(2,I)+2*L(3,I)+H*G(I))/6
```

INTEGX (continued)

```
830 NEXT I
840 T=INT(T*100000+.5)/100000
850 H2=H2+1
860 IF H2<H1-.5 THEN 970
870 IF T<T2 THEN 970
880 H2=0
890 J=J+1
900 S(J,1)=T
910' PRINT T;
920 FOR I=1 TO N2
930' PRINT TAB(15*I);X(N(I));
940 S(J,I+1)=X(N(I))
950 NEXT I
955 PRINT#2:T;X(I8+1)
960' PRINT
970 IF T>T1 THEN 1090
980 GO TO 630
990 READ H,T2,T1,M
991 N=I8
1010 READ H1
1020 FOR I=1 TO 4
1030 READ N(I)
1040 NEXT I
1045 N(1)=I8+1
1050 FOR I=1 TO M
1060 READ I(I),E$(I)
1070 NEXT I
1080 GO TO 460
1090 PRINT
1095 PRINT#2:" 1E37 , 1E37 "
1100 IF M=0 THEN 1230
1110 FOR I=1 TO 15*(N2+1)
1120 PRINT "-";
1130 NEXT I
1140 PRINT
1150 PRINT
1160 PRINT "VARIABLE","SYMBOL"
1170 PRINT
1180 FOR I=1 TO M
1190 PRINT "X(";Z(I);")",TAB(17);E$(I)
1200 NEXT I
1210 PRINT
1230 PRINT
1240 FOR I=1 TO 72
1250 PRINT "=";
1260 NEXT I
1270 PRINT
1280 GO TO 1370
1290 REM ****** SUBROUTINE TO EVALUATE THE G(I)'S ***********
1292 FOR I=1 TO I8-1
```

INTEGX (continued)

```
1293 G(I) = X(I+1)
1294 NEXT I
1296 G(I8) = 1.
1298 FOR I=1 TO I8
1299 G(I8)=G(I8) - D(I)*X(I)
1302 NEXT I
1305 X(I8+1)=0.
1306 FOR I=1 TO I7
1307 X(I8+1)=X(I8+1) + Q(I)*X(I)
1309 NEXT I
1320 RETURN
1330 DATA .001,0,20,0
1352 DATA 50,1,0,0,0
1360 DATA 1,"*",2,"#",3,"+",4,"@"
1370 END
```

APPENDIX D. COMPUTER PROGRAM FOR THE TIME SOLUTION OF THE SYSTEM EQUATIONS.

The program on the next page solves the set of equations

$$\dot{\underline{x}} = A\underline{x} + B\theta$$

$$e_o = C_4\underline{x}$$

$$\underline{z} = F\underline{z} + B\theta + \underline{K}e_o$$

$$\hat{e} = Mz$$

The equations included are those of the ideal observer and rate sensor.

INTEG

```
100' INTEG
110'
120' DESCRIPTION--Solves a set of N first-order differential equations
130' of the form:
140'
150'       dX(I) = G(I) for I = 1,2,.......,N,
160'
170' when the initial conditons are known, by the fourth-order Runge Kutta
180' technique and provides a plot of T vs X(N1), X(N2), X(N3), and
190' X(N4) in that order depending on selection in DATA statement.
200'
210' INSTRUCTIONS--The functions, G(I), are to be inserted after line 1350
220' and before line 300 in the following form:
230'
240'        1350   G(1) = WHATEVER IT IS
250'        1360   G(2) = WHATEVER IT IS
260'
270' The entire mathematical model should go in this section including
280' algebraic constraints.
290'
300'        1520 DATA N,H,T2,T1,M
310'        1530 DATA X(1),X(2),.....,X(N)
320'        1540 DATA H1,N1,N2,N3,N4
330'
340' Where N = Number of dependent variables
350'       H = Increment size
360'       T2 = Initial value of T
370'       T1 = Final value of T
380'       M = Number of variables to be plotted
390'       H1 = No. of increments of H between print outs (must be an
400'            integer.)
410'       N1,N2,N3,N4 identify the dependent variables to be printed out.
420'
430'       In line 1530 X(1)....X(N) = Initial values of X(1)...X(N). In
440'       general, X(1)....X(N) = Dependent variables.
450'
460'       T = Independent variable
470'       G(I) = Known functions = dX(I)/dT
480'
490'    *    *    *    *      MAIN PROGRAM      *    *    *    *
500 LIBRARY "L.ES***:PLOT1"
510 DIM X(50),G(50),L(3,50),S(500,6)
520 FOR I=1 TO 72
530 PRINT "=";
531 NEXT I
532 PRINT"PLOT FILE NAME"
533 INPUT T$
534 FILE#2: T$
535 SCRATCH#2
550 PRINT
```

INTEG (continued)

```
560 GO TO 1100
570 IF M=0 THEN 640
580 PRINT "PLOT SYMBOL";
590 FOR I=1 TO M
600 PRINT TAB(15*I+3);E$(I);
610 NEXT I
620 PRINT
630 PRINT
640 PRINT "T";
650 FOR I=1 TO 4
660 IF N(I)=0 THEN 710
670 PRINT TAB(15*I);"X(";N(I);")";
680 Z(I)=N(I)
690 N2=N2+1
700 NEXT I
710 PRINT
720 IF T<T2 THEN 1080
730 GO TO 1000
740 GO SUB 1360
750 FOR I=1 TO N
760 L(1,I)=H*G(I)
770 X(I)=X(I)+L(1,I)/2
780 NEXT I
790 T=T+H/2
800 GO SUB 1360
810 FOR I=1 TO N
820 L(2,I)=H*G(I)
830 X(I)=X(I)-L(1,I)/2+L(2,I)/2
840 NEXT I
850 GO SUB 1360
860 FOR I=1 TO N
870 L(3,I)=H*G(I)
880 X(I)=X(I)-L(2,I)/2+L(3,I)
890 NEXT I
900 T=T+H/2
910 GO SUB 1360
920 FOR I=1 TO N
930 X(I)=X(I)-L(3,I)+(L(1,I)+2*L(2,I)+2*L(3,I)+H*G(I))/6
940 NEXT I
950 T=INT(T*100000+.5)/100000
960 H2=H2+1
970 IF H2<H1-.5 THEN 1080
980 IF T<T2 THEN 1080
990 H2=0
1000 J=J+1
1010 S(J,1)=T
1020 PRINT T;
1030 FOR I=1 TO N2
1040 PRINT TAB(15*I);X(N(I));
1050 S(J,I+1)=X(N(I))
```

INTEG (continued)

```
1060 NEXT I
1065 PRINT#2:T;X(10)
1070 PRINT
1080 IF T>T1 THEN 1200
1090 GO TO 740
1100 READ N,H,T2,T1,M
1110 MAT READ X(N)
1120 READ H1
1130 FOR I=1 TO 4
1140 READ N(I)
1150 NEXT I
1160 FOR I=1 TO M
1170 READ I(I),E$(I)
1180 NEXT I
1190 GO TO 570
1200 IF M=0 THEN 1320
1210 FOR I=1 TO 15*(N2+1)
1220 PRINT "-";
1230 NEXT I
1240 PRINT
1250 PRINT "VARIABLE","SYMBOL"
1260 PRINT
1270 FOR I=1 TO M
1280 PRINT "X(";Z(I);")",TAB(17);E$(I)
1290 NEXT I
1300 PRINT
1320 FOR I=1 TO 72
1330 PRINT "=";
1340 NEXT I
1350 GO TO 1560
1360 K1=.0253
1370 K1=.0253
1380 K2=.04112
1390 K3=.984566
1400 K4=-167.3
1410 G(1)=X(2)
1420 G(2)=X(3)
1430 G(3)=X(4)
1440 G(4)=-1023.4*X(1)-4738.4*X(2)-15653*X(3)-175.3*X(4)+1170*X(9)
1450 G(5)=K1*(X(4)-X(8))+X(6)
1460 G(6)=K2*(X(4)-X(8))+X(7)
1470 G(7)=K3*(X(4)-X(8))+X(8)
1480 G(8)=K4*(X(4)-X(8))-1023.4*X(5)-4738.4*X(6)-15653*X(7)-175.3*X(8)+1170*X(9)
1490 G(9)=1
1500 X(10)=13.33*X(8)+4.04*X(7)+.87*X(6)
1510 RETURN
1520 DATA 10,.005,0,20,0
1530 DATA 0,0,0,0,0,0,0,0,0,0
1540 DATA 10,10,8,0,0
1550 DATA 1,"*",2,"#",3,"+",4,"@"
     END
```

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